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APPLICATIONS FOR ADMISSION AND TRANSFER

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CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
Report of the Committee of Engineering Foundation on Arch Dam Investigation	May, 1928	
Discussion	Nov., 1928, Jan., April, May, Aug., 1929, May, 1930, Mar., 1931	Closed
Effect of Turbulence on the Registration of Current Meters. *David L. Yarnell and Floyd A. Nagler*	Dec., 1929	
Discussion	Feb., May, Aug., Sept., Nov., 1930, Mar., 1931	Closed
Plastic Flow in Concrete Arches. *Lorenz G. Straub*	Jan., 1930	
Discussion	April, May, Aug., Sept., Dec., 1930	Closed
Relation Between Rail and Waterway Transportation: A Symposium	Mar., 1930	
Discussion	April, May, Aug., Sept., Dec., 1930, Jan., 1931	Closed
Analysis of Continuous Frames by Distributing Fixed-End Moments. *Hardy Cross*	May, 1930	
Discussion	Sept., Oct., Nov., 1930, Jan., Feb., Mar., 1931	Uncertain
The Training Wall Across the Liao Bar in Manchuria. *P. N. Fawcett*	May, 1930	
Discussion	Sept., Oct., Dec., 1930	Closed
Evaporation as a Function of Insolation. *Burt Richardson*	May, 1930	
Discussion (Author's closure)	Sept., Oct., 1930, Mar., 1931	Closed
City Planning as Related to the Smaller Cities. *E. A. Wood*	May, 1930	
Discussion	Aug., Sept., 1930	Mar., 1931
Silting and Life of Southwestern Reservoirs. *R. G. Hemphill*	May, 1930	
Discussion	Feb., 1931	Mar., 1931
Highways as Elements of Transportation. *Fred Lavis*	Aug., 1930	
Discussion	Oct., Dec., 1930	Mar., 1931
Some Aspects of Water Conservation. *R. A. Sutherland*	Sept., 1930	
Discussion	Jan., Feb., Mar., 1931	Mar., 1931
Construction of La Ola Pipe Line in Chile. *W. B. Saunders*	Sept., 1930	
Discussion	Feb., 1931	Mar., 1931
Tests of Broad-Crested Weirs. *James G. Woodburn*	Sept., 1930	
Discussion	Oct., Dec., 1930, Feb., 1931	Mar., 1931
Stresses in Gravity Dams by Principle of Least Work. *B. F. Jakobsen*	Sept., 1930	
Discussion	Sept., Nov., 1930, Jan., Feb., 1931	Mar., 1931
Theory of Similarity and Models. *Benjamin F. Groat*	Oct., 1930	
Discussion	Jan., Feb., Mar., 1931	Mar., 1931
Formulas for Rainfall Intensities of Long Duration. *Merrill M. Bernard.*	Oct., 1930	
Discussion	Feb., 1931	Mar., 1931
Progress Report of the Committee of the Sanitary Engineering Division on Filtering Materials for Water and Sewage Works.	Oct., 1930	
Discussion	Oct., 1930, Mar., 1931	Mar., 1931
The Buttressed Dam of Uniform Strength. *Herman Schorer.*	Nov. 1930	
Discussion	Jan., Feb., Mar., 1931	Mar., 1931
Economical Design of Hydraulic Pipe-Line Dredge. *John Francis Cushing.*	Nov., 1930	
Discussion	Feb., Mar., 1931	Mar., 1931
Highway Location: General Considerations. *Gibb Gilchrist*	Nov., 1930	
Discussion	Feb., 1931	Mar., 1931
Highway Location: Practical Considerations. *A. R. Losh*	Nov., 1930	
Discussion	Feb., 1931	Mar., 1931
Relocating Land Surveys in Texas. *B. F. Williams*	Nov., 1930	Mar., 1931
Flow of Water in Tidal Canals. *Earl I. Brown*	Dec., 1930	
Discussion	Feb., Mar., 1931	Apr., 1931
The Don Martin Project. *Andrew Weiss*	Dec., 1930	
Discussion	Mar., 1931	Apr., 1931
Design of a Reinforced Concrete Skew Arch. *Bernard L. Weiner.*	Jan., 1931	
A Discharge Diagram for Uniform Flow in Open Channels. *Murray Blanchard*	Jan., 1931	May, 1931
Legal Problems Involved in Establishing Set-Back Lines. *Clifton Williams.*	Jan., 1931	May, 1931
The Eight-Mile Cascade Tunnel, Great Northern Railway: A Symposium.	Feb., 1931	Aug., 1931
Measuring Irrigation Deliveries in the Panjab. *E. S. Lindley*	Feb., 1931	Aug., 1931
Surveys for Swift River Reservoir of the Boston Metropolitan Water Supply. *N. LeRoy Hammond*	Feb., 1931	Aug., 1931

CONTENTS FOR MARCH, 1931

PAPERS

PAGE

Flow of Ground-Water as Applied to Drainage Wells.
By M. R. Lewis, M. Am. Soc. C. E. 411

Gravity Dams Arched Down Stream.
By B. J. Lambert, M. Am. Soc. C. E. 425

DISCUSSIONS

Report of the Committee of Engineering Foundation on Arch Dam Investigation.
By Eugene Kalman, M. Am. Soc. C. E. 431

Effect of Turbulence on the Registration of Current Meters.
By Forrest Nagler, M. Am. Soc. C. E. 461

Analysis of Continuous Frames by Distributing Fixed-End Moments.
By Francis P. Witmer, M. Am. Soc. C. E. 469

Evaporation as a Function of Insolation.
By Burt Richardson, Esq. 475

Some Aspects of Water Conservation.
By John W. Pritchett, Assoc. M. Am. Soc. C. E. 477

Theory of Similarity and Models.
By Ed S. Smith, Jr., Esq. 483

The Buttressed Dam of Uniform Strength.
By A. A. Eremin, Assoc. M. Am. Soc. C. E. 487

Economical Design of Hydraulic Pipe-Line Dredge.
By Arthur L. Collins, Assoc. M. Am. Soc. C. E. 491

Flow of Water in Tidal Canals.
By Messrs. G. T. Rude, Eugene E. Halmos, and W. M. Black. 495

The Don Martin Project.
By C. H. Howell, M. Am. Soc. C. E. 503

Progress Report of the Committee of the Sanitary Engineering Division on Filtering Materials for Water and Sewage Works.
By Messrs. W. S. Mahlie, and O. J. Ripple. 507

CONTENTS FOR MARCH, 1931 *(Continued)*

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

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SOCIETY AFFAIRS

Annual Report of the Board of Direction for the Year Ending December 31, 1930 . . . . . following page 510

MEMBERSHIP

Applications for Admission and Transfer. . . . . following page 24

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

FLOW OF GROUND-WATER AS APPLIED TO DRAINAGE WELLS

By M. R. Lewis,[1] M. Am. Soc. C. E.

Synopsis

Much study has been given to the capacity of wells to furnish water and oil, and, with the use of deep wells for the drainage of irrigated lands, the shape and extent of the cone of depression become of great importance.

In this paper three types of wells are discussed: (*A*) Artesian wells (not necessarily flowing), with a perforated casing extending through the water-bearing stratum; (*B*) wells in which the water-table is in the water-bearing stratum and which penetrate its full depth and (*C*) open-bottom wells, that is, wells which just reach the water-bearing stratum.

Cases in which the zone of influence is either definite or indefinite are considered for each type; and formulas for the draw-down curves of each of these types of wells are derived.

The conclusion is drawn that if the area to be drained is more than a few hundred feet in diameter, successful drainage will depend on a general lowering of the water-table.

To secure this lowering, wells should be designed to have the greatest possible capacity with an economical lift. Such capacity may be secured most readily, where conditions are favorable, by deep wells.

Introduction

Much study has been given to the theory of the flow of ground-water and oil into wells. Practically all this study has been directed toward estimating the quantity of water or oil which might be secured from wells under conditions differing as to diameter and depth, thickness and character of water-bearing strata, and depth of draw-down. Since wells and deep-well pumps have been used for the drainage of irrigated lands, the shape and extent of the draw-down curve or cone of depression have become of primary importance.

It is difficult, if not impossible, to determine by field studies and laboratory tests the exact capacity for the transmission of water of the water-bearing

Note.—Written discussion on this paper will be closed in August, 1931, *Proceedings*.

[1] Irrig. and Drainage Engr., Oregon State Agri. Coll., and Div. of Agri. Eng., U. S. Dept. of Agriculture, Corvallis, Ore.

strata in any given case. In spite of this fact, however, it is well worth while to consider the effect on the draw-down curve of changes in the various controlling factors under certain assumed conditions.

The first assumption which is made in such studies is that the water-bearing strata are uniform. This will be approximately true in most cases, but far from correct in others. Since the variations in this regard may be infinite in number, and are impossible of determination, it does not seem worth while to attempt a discussion of such cases.

A second assumption, which is applied in the case of those wells that penetrate the full depth of the water-bearing stratum, is that the water pressure is transmitted without hindrance in a vertical direction and that it corresponds to static pressure in any given vertical line; in other words, that the water would rise to the same level in each of a series of piezometer tubes open to the ground-water at various elevations below the water-table in the same vertical. This assumption is probably true in regard to artesian wells that extend through a water-bearing stratum, when the retaining beds are impervious. When the water-table is in the water-bearing stratum, the lines of flow will not be horizontal, and the second assumption is not correct. However, only in the case of wells in which the draw-down is large as compared with the depth of the water-bearing strata—and even then only near the well where the slope of the water-table is great—will the error be appreciable.

In this paper the piezometric surface and water-table are assumed to be level before pumping commences. From this assumption it follows that there is no flow of the ground-water except to the well.

In the case of the drainage wells designated herein by the secondary letter, *b*, the assumption that the quantity of water reaching the water-table is uniform per unit of land area, seems to require either that the well under consideration is in the interior of a homogeneous group of wells the zones of influence of which are contiguous, or that the water-table at the boundary of the zone of influence is at the surface of the ground.

It is evident that these assumptions in themselves limit the rigorous application of the formulas to very special cases. An exact application of the laws of hydrodynamics would require consideration of the effect of inertia and of the probable inapplicability of the Darcy law to extremely high velocities. The writer has made no attempt to do this. It is believed, on the other hand, that the general relationship between the variables considered are correctly presented and have rather wide application.

The fact that the cost of power is a controlling factor in irrigation pumping makes the total extent of the draw-down in irrigation wells a matter of first consideration. In the case of drainage wells, the extent of the draw-down at the wells and throughout the drained area is of primary importance. The cost of power (except for variations in demand charges, pump and motor efficiencies, and friction losses in the piping) is proportional to the total height through which water must be lifted. This total lift is made up of the distance the water must be lifted above the static water-table, plus the total draw-down. The former is fixed by the topography and other considerations, but is not

affected by changes in the well. The total draw-down is made up of the friction in the well tube between the point where the water enters the well and the entrance to the suction pipe of the pump, plus the loss of head at the perforated well casing or strainer, if one is used, or the entrance head into the well tube, plus the loss of head through the water-bearing strata. Under normal conditions the last item is by far the largest part of the total draw-down and is the only part considered herein. The loss of head at the strainer is known to have been an important factor in many instances.

The shape of the draw-down curve will depend on the size and character of the well and on its relation to the water-bearing strata and the source of water supply. Theoretical formulas and curves for three types of wells are given herein. For each type the effect of a local or distant water supply is taken up. Furthermore, the effect of changes in the character and thickness of the water-bearing strata, of the diameter and depth of the well, and of the quantity of water pumped is illustrated.

Notation

In addition to the symbols, defined by diagrams in Figs. 1, 4 and 6, the following notation is adopted for use in this paper and its discussion:

C = a constant.
d = diameter of sand particles, in millimeters.
k = transmission coefficient in water-bearing strata.
k_1 = transmission coefficient in relatively impervious overlying stratum.
K = a factor that varies with the porosity of the material.
Q = quantity of water passing concentric surfaces of radius, r.
Q_1 = the discharge of the well.
V = velocity of a stream of water.
w = the rate of downward percolation.

In order to facilitate reference, various types of wells are identified by symbols in the paper. Briefly, the definitions of these symbols are, as follows:

Type *Aa* is an artesian well with a perforated casing that extends through the water-bearing stratum. This stratum is assumed to lie between two impervious retaining beds and contains water under such hydrostatic pressure that the water level just outside the casing will not be lowered, by pumping, below the upper edge of the stratum.

Type *Ab* is an artesian well with a perforated casing that extends through the water-bearing stratum. In this case the overlying stratum is only relatively impervious and the supply to the well is derived by vertical flow, or percolation.

Type *B* consists of wells in which the water-table is in the water-bearing stratum and which penetrate the full depth of the stratum.

Type *C* is an open-bottom well which just reaches the water-bearing stratum.

The relations between Types *Ba* and *Bb* and between Types *Ca* and *Cb* are similar to that between Types *Aa* and *Ab*.

TYPE *A*.—ARTESIAN WELLS EXTENDING THROUGH WATER-BEARING STRATUM

The case of the simplest well (Type *Aa*) has been discussed by Slichter,[2] Turneaure,[3] and others. It is that of a well penetrating the full depth of a water-bearing stratum lying between two relatively impervious retaining beds and containing water under such hydrostatic pressure that the water level in the well, or rather just outside the well-casing, will not be lowered, by pumping, below the upper edge of the water-bearing stratum. In each case these writers

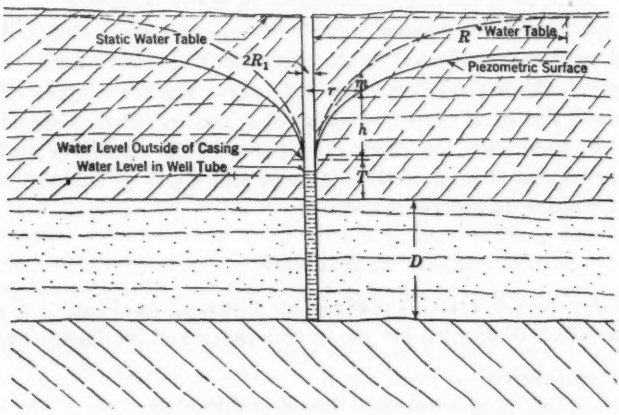

FIG. 1.—TYPE *A* WELL.

have assumed that all the water came into the stratum from outside the zone of influence of the well. Under such conditions, the curve of the modified piezometric surface would rise indefinitely, but at a decreasing rate, as the distance from the well is increased. The water flowing toward the well will pass through concentric cylindrical surfaces in the water-bearing stratum, having areas of $2\pi D r$ (for notation, see Fig. 1). The velocity of flow according to the well-known law of Darcy will be proportional to the slope of the hydraulic gradient. For convenience, the transmission coefficient, k, used by Slichter,[2] and defined as the quantity of water passed in unit time by a prism of the material of unit cross-sectional area and unit length under the pressure of a column of water of unit height—using as units, the foot and the minute—has been adopted for this paper. The velocity of a hypothetical stream of water occupying the full area of the cross-section at any distance from the well is, therefore,

$$V = k\frac{dh}{dr} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The quantity, Q, flowing toward the well, is equal to the discharge of the well, Q_1, and is constant for all values of r; therefore,

$$Q = Q_1 = 2\pi D r k\frac{dh}{dr} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

[2] "Theoretical Investigation of the Motion of Ground Waters", by C. S. Slichter, U. S. Geological Survey, Nineteenth Annual Rept., Pt. 2, p. 323, Washington, D. C., 1899.

[3] "Public Water Supplies", by F. E. Turneaure and H. L. Russell, Third Edition, p. 262, John Wiley & Sons, New York, 1927.

Integrating,

$$h = \frac{Q_1}{2\,\pi\,D\,k} \log_e r + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

When $r = R_1$, $h = 0$; and if $R_1 = 1$, $C = 0$. If R_1 has any other value, C will vary with changes in the values of the other constants.

On Fig. 2 the draw-down curves for certain assumed values of Q_1, D, R_1, and k are shown.

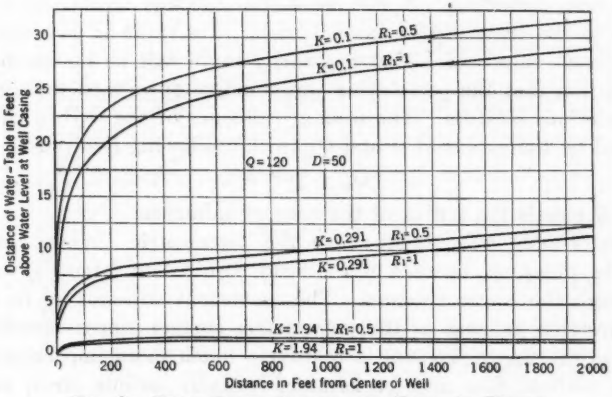

FIG. 2.—DRAW-DOWN CURVES FOR TYPE *Aa* WELLS.

Type Ab.—Where wells are used for the drainage of irrigated lands during the irrigation season, or of humid lands during rainy seasons, the conditions are modified to the extent that the water supply does not come along horizontal planes from an indefinite distance, but downward from the soil overlying the cone of depression.

When a well is first put into operation the water withdrawn from the cone of depression will most probably reach the water-bearing stratum at a greater rate per unit of land area near the well than at a greater distance. After the water-table has been lowered to an equilibrium position the supply will come entirely from the surface. The rate of downward percolation in practically all cases will be governed by surface conditions and will be substantially independent of the draw-down. For this reason a uniform rate is assumed.

If w is the rate of downward percolation, in cubic feet per minute per square foot of horizontal area, the conditions are expressed by the following formulas:

$$Q = Q_1 - w\,\pi\,r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, Q_1 is the discharge of the well and Q, the variable quantity passing the concentric cylindrical surfaces of the radius, r. Substituting for Q in Equation (4), its equivalent in Equation (2):

$$Q_1 = 2\,\pi\,D\,r\,k\,\frac{dh}{dr} + w\,\pi\,r$$

$$Q_1\,dr = 2\,\pi\,D\,r\,k\,dh + w\,\pi\,r^2\,dr$$

$$dh = \frac{Q_1}{2\,\pi\,D\,r\,k}\,dr - \frac{w\,r}{2\,D\,k}\,dr$$

and,

$$h = \frac{Q_1}{2\,\pi\,D\,k}\log_e r - \frac{w}{4\,D\,k}\,r^2 + C \quad \ldots\ldots(5)$$

If w is assumed as any probable value; as, for example, 0.000017 (3.67 ft. in depth over the surface in 150 days), and if $R_1 = 1$ ft., the values of C will be practically zero. With other values of R_1, C will be the same in this case as in the one preceding. It will be noted that the factor, w, is somewhat analogous to the transmission coefficient, k. Its value is so small compared to the common values of k that ordinarily there will be no inconsistency in the assumption that the percolating water moves downward through the relatively impervious stratum. The zone of influence of the well in this case will be governed by the values of w and Q_1 in the following relation:

$$Q_1 = w\,\pi\,R^2 \quad \ldots\ldots(6)$$

in which, R equals the radius of the zone of influence.

The water-table will stand above the piezometric surface, as both are depressed by pumping, because of the head required to force the water downward through the upper stratum. The movement will not be in an exactly vertical direction as long as the piezometric surface slopes toward the well. However, as the upper stratum is relatively much more impervious than the lower one, vertical flow may be assumed without serious error, except very close to the well where the slope of the water-table is extremely steep. Let k_1 represent the transmission coefficient for the overlying stratum; m, the height of the water-table above the piezometric surface; and T, the thickness of the more impervious material below the level of the piezometric surface at the well. Under these conditions the following formulas will hold:

$$w = \frac{m}{m + h + T}\,k_1 \quad \ldots\ldots(7)$$

$$m = \frac{h + T}{\frac{k_1}{w} - 1} \quad \ldots\ldots(8)$$

and,

$$m + h = \frac{h\,k_1}{k_1 - w} + \frac{w\,T}{k_1 - w} \quad \ldots\ldots(9)$$

Substituting in Equation (9) the value of h given in Equation (5),

$$m + h = \left(\frac{Q_1}{2\,\pi\,D\,k}\log_e r - \frac{w\,r^2}{4\,D\,k} + ,C\right)\frac{k_1}{k_1 - w} + \frac{w\,T}{k_1 - w} \quad \ldots\ldots(10)$$

Equations (5) and (10) will hold only for values of r equal to, or less than, R. (See Fig. 3 for typical results.)

Type *B*.—Wells in Which Water-Table Is in Water-Bearing Stratum

Another common case (Type *Ba*) is illustrated in Fig. 4. The water-table, both under the static condition and during pumping, lies wholly within the water-bearing stratum. Turneaure[3] has shown that the formula for the draw-

down curve in this case is (adopting the notation used herein and shown in Fig. 4):

$$h^2 = \frac{Q_1}{\pi\, k} \log_e r + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

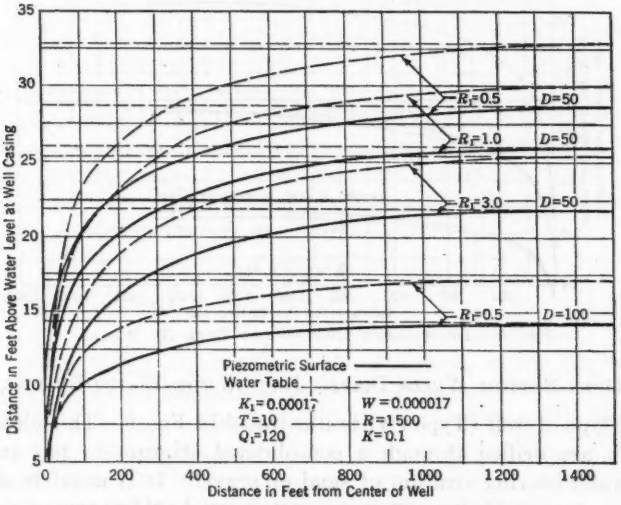

FIG. 3.—DRAW-DOWN CURVES FOR TYPE *Ab* WELLS.

In Equation (11), when $r = R_1$, $h = D$; and, if $r = R_1 = 1$, then $C = D^2$. Here, again, the value of C will vary with the other factors if R_1 does not equal 1.

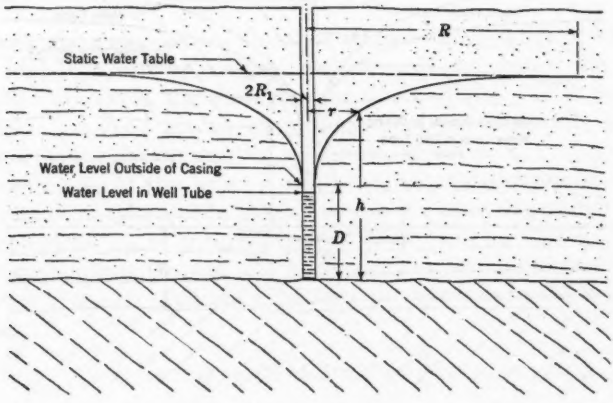

FIG. 4.—TYPE *B* WELL.

Type Bb.—In the more typical case of drainage wells, where the water supply comes from the surface above the cone of depression:

$$h^2 = \frac{Q_1}{\pi\, k} \log_e r - \frac{w\, r^2}{2\, k} + C \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Again, when $r = R_1$, $h = D$; and if $r = R_1 = 1$, the second term on the right of the equation is practically zero and $C = D^2$. Fig. 5 shows some typical draw-down curves for this case.

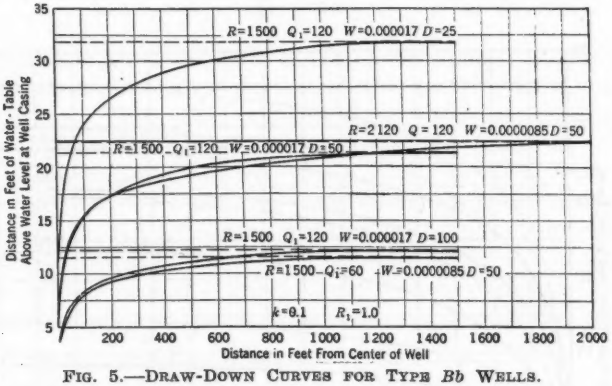

FIG. 5.—DRAW-DOWN CURVES FOR TYPE *Bb* WELLS.

TYPE *C*.—OPEN-BOTTOM WELLS DRIVEN JUST TO THE WATER-BEARING STRATUM

A third type of well (Type *Ca*) is illustrated in Fig. 6. This class includes wells which are drilled through a consolidated stratum to tap an artesian flow in a water-bearing stratum of sand or gravel. It is usual in developing wells of this type to pump out more or less sand. If the overlying bed is strong enough to stand up, the resulting cavity in the sand below will be cone-shaped with the point downward. The slope of the sides will be that of

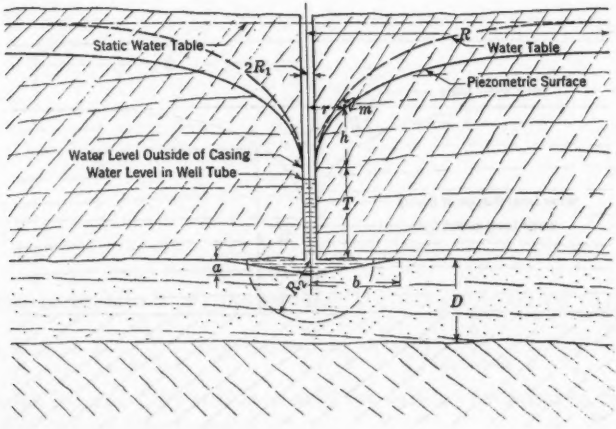

FIG. 6.—TYPE *C* WELL.

the angle of repose of the material when supersaturated with water. Observation of sand which is discharging water when excavated below the water-table indicates that the slope is about 1 on 12. This cavity will increase in size until the velocity of the water leaving the surface of the sand is just

insufficient to lift the finer particles. In a mixed material the finer particles will be washed out, leaving the cavity lined with the coarser particles. For this reason it is not necessary in all cases to excavate a shaft of large diameter to secure the maximum efficiency of this type of well. The velocity required to lift a particle of sand will vary, of course, with the size of the particle.

It is not necessary that the velocity of the water in the cavity just above the sand be great enough to carry the sand particles vertically upward. The bubbling up of sand in a spring shows the action which takes place. Richards,[4] in studying ore-classifying machinery, determined the velocity of the rising streams which would hold quartz particles of various sizes in full "teeter". This condition is somewhat similar to that which takes place in wells or springs. By plotting them on logarithmic paper his data are found to be represented with reasonable accuracy by the following formulas:

For particles 1.25 to 7.0 mm. in diameter:

$$V = 5.5\, d^{0.75} \quad \ldots\ldots (13)$$

For particles 0.25 to 1.25 mm. in diameter:

$$V = 5.1\, d^{1.15} \quad \ldots\ldots (14)$$

in which, V equals the velocity of the rising stream, in feet per minute, and d is the diameter of the quartz particles, in millimeters.

The radius, b, in feet, of the cavity in the sand stratum for the two grades of sand will then be as follows:

For particles of 1.25 to 7.0 mm.:

$$b = \frac{{Q_1}^{0.5}}{4.16\, d^{0.38}} \quad \ldots\ldots (15)$$

For particles of 0.25 to 1.25 mm.:

$$b = \frac{{Q_1}^{0.5}}{4.01\, d^{0.58}} \quad \ldots\ldots (16)$$

Whenever the water-bearing material is made up of sands of various sizes, the finer materials will be removed entirely, and the size of the cavity will undoubtedly depend on the proportion of fine material. It is probable that the condition assumed herein, of uniform-sized particles, will be the most unfavorable for the development of large flows of water.

For the purpose of this paper it will be sufficiently accurate to assume that the cavity is hemi-spherical in shape and has the same superficial area as the cone. The radius, R_2, of this hypothetical cavity will be $0.716\, b$. In this case the water approaching the well may be assumed to pass hemi-spherical surfaces having areas of $2\, \pi\, r^2$. Following the same procedure as in the case of wells of other types,

$$Q = Q_1 = 2\, \pi\, r^2\, k\, \frac{dh}{dr} \quad \ldots\ldots (17)$$

and,

$$h = -\frac{Q_1}{2\, \pi\, k\, r} + C \quad \ldots\ldots (18)$$

[4] "Development of Hindred-Settling Apparatus", by Robert H. Richards, *Transactions*, Am. Inst. Min. and Met. Engrs., Vol. 41, p. 425, 1910.

If $r = R_2$, $h = 0$,

$$C = \frac{Q_1}{2\,\pi\,k\,R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Equation (18) will hold only if the water-bearing stratum is of very great depth. If this stratum is of moderate depth, an approximate solution may be reached by using Equation (18) for values of r between 0 and D and Equation (3) for values of r greater than D.

Type Cb.—For the drainage well there will again be an approximation if the term is subtracted that accounts for water reaching the water-table from the surface; thus,

$$Q = Q_1 - w\,\pi\,r^2$$

$$Q_1 = 2\,\pi\,r^2\,k\,\frac{dh}{dr} + w\,\pi\,r^2$$

$$dh = \frac{Q_1}{2\,\pi\,k\,r^2}\,dr - \frac{w}{2\,k}\,dr$$

and,

$$h = -\frac{Q_1}{2\,\pi\,k\,r} - \frac{w\,r}{2\,k} + C \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Again, the second term on the right is negligible for small values of r and practically so for all values.

The same relation between the piezometric surface and the water-table holds in this case as in that of wells of Type *Ab*. (See Fig. 7 for typical curves.) The height of the water-table will be:

$$m + h = \left(-\frac{Q_1}{2\,\pi\,k\,r} - \frac{w\,r}{2\,k} + C\right)\frac{k_1}{k_1 - w} + \frac{w\,T}{k_1 - w} \quad \ldots\ldots\ldots (21)$$

Another type of well which has been used in some faint-hearted attempts at drainage, and one that is fairly common in the irrigation field, is the dug well which just reaches into the water-table and is pumped to its utmost capacity. The draw-down curves for wells of this type would be quite similar to those of large diameter wells of Type *C*. The flow of water through the stratum above the level of the bottom of the well would tend, of course, to make the curve flatten out faster as r is increased.

Permeability of Water-Bearing Strata

Many experimenters have worked on the problem of the rate at which soils, sands, and gravels will transmit water. Much of this study has been directed toward establishing some formula which would make possible the accurate prediction of permeability from an analysis of the physical characteristics of the material. Thus far no formula satisfactory over a wide range of materials and conditions has been found. The one devised by Slichter[5] is probably as satisfactory as any. It is as follows:

$$k = (1.1846)\,\frac{d^2}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

[5] "Theoretical Investigation of the Motion of Ground-Waters" U. S. Geological Survey, Nineteenth Annual Rept., Pt. 2, p. 322, Wash., 1899; "The Motions of Underground Waters", U. S. Geological Survey, *Water Supply Paper 67*, Wash., 1902; "Field Measurements of the Rate of Movement of Underground Water", U. S. Geological Survey, *Water Supply Paper 140*, Wash., 1905.

in which, k is the transmission coefficient, in cubic feet per minute, per square foot of cross-section, per foot of head, through a column, 1 ft. long. The number, (1.1846), is the common logarithmic of a constant factor or coefficient. Factor K varies with the porosity of the material. Equation (21) was used to determine the value of k corresponding to the given effective sizes and porosities in the computations for wells of Type *C*.

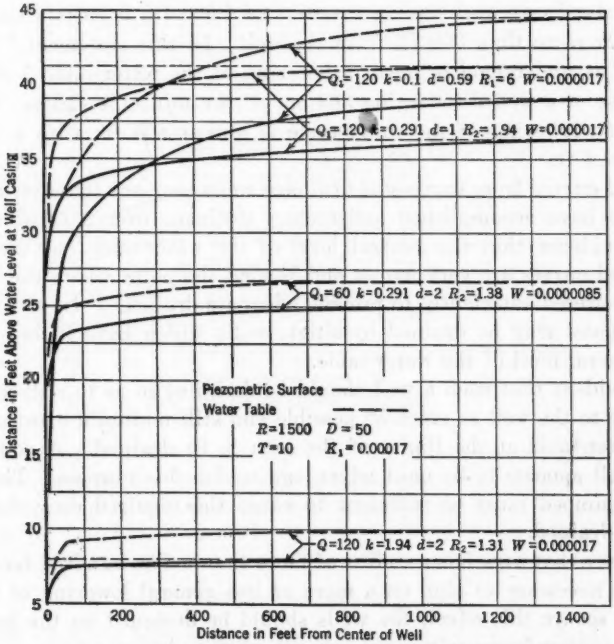

FIG. 7.—DRAW-DOWN CURVES FOR TYPE *Cb* WELLS.

Hazen,[6] Slichter,[5] Stearns,[7] and others, have given the results of experimental determinations of the rate of flow through sands and gravels. It is difficult to secure samples, for laboratory test, of unconsolidated material occurring near the surface in such a manner as not to disturb its structure and porosity. Such samples cannot be secured of the deeper-lying strata. In spite of this difficulty laboratory tests of the rate of flow seem to be much more satisfactory than the attempt to compute the rate from mechanical analyses.

Meinzer[8] refers to a method devised by Thiem, which appears to promise good results. In this method use is made of the slope of the water-table toward a well which is being pumped.

[6] "Some Physical Properties of Sands and Gravels with Special Reference to Their Use in Filtration", by the late Allen Hazen, M. Am. Soc. C. E., Massachusetts State Board of Health, Twenty-fourth Annual Rept., 1893.

[7] "Laboratory Tests on Physical Properties of Water-Bearing Materials", by Nora D. Stearns, U. S. Geological Survey, *Water Supply Paper 596 F.*, Wash., 1927.

[8] "Methods of Estimating Ground-Water Supplies, Pt. I": Contribution to a Symposium, by O. E. Meinzer, Annual Technical Session, Soc. of Economic Geologists, 1928.

Application of Curves

The most obvious conclusion to be drawn from the curves and data shown herein is that satisfactory drainage by wells depends on the removal of sufficient water to lower the general ground-water level. This, of course, agrees with the general opinion of engineers engaged in the drainage of irrigated lands by open ditches and tile-drains. It will be noted that in none of the plotted curves in which the wells are taking all the water supplied to the ground-water in an area having a radius of 1 500 ft., is satisfactory drainage secured for more than 300 ft. from the well. In this statement "satisfactory drainage" is understood to mean a lowering of the water-table 5 ft., or more. In the case of a well discharging 360 cu. ft. per min., the radius of the circle with a 5-ft. lowering of the water-table is almost 800 ft., with a total draw-down of 31.4 ft.

Plotted curves from successful drainage wells bear out these results. Where such wells have accomplished satisfactory drainage over a considerable area there is evidence that the general level of the water-table has been affected. The plotted curves are very flat at and beyond the point where the water-table has been lowered sufficiently to provide adequate drainage. In some instances, local pot-holes may be drained by single wells which have little or no effect on the general level of the water-table.

It is evident that such a well should be designed so as to reduce the draw-down close to the well as much as possible and still maintain a sufficient depth to the water-table at the limits of the area to be drained. A comparatively shallow well appears to be most advantageous for this purpose. The quantity of water pumped must be sufficient to cause the required draw-down in the area to be drained.

Whenever that area has a radius of more than a few hundred feet, it would seem to be necessary to plan on a more or less general lowering of the water-table. To secure this effect, the wells should be designed on the basis of the maximum economic capacity.

Inspection of the formulas and of the resulting curves shows that in each class of wells the quantity of water delivered is proportional to the transmission coefficient. An increase in the depth of water-bearing strata penetrated is followed by a proportional increase of flow in Type *A* wells and deeper Type *B* wells. In shallow wells of the latter type the effect will depend on the relation between the depth of draw-down and the total depth of the well. In wells of Type *A* and Type *B* a change in the diameter of the well has a comparatively small effect on the capacity.

Note in Fig. 3 that the total draw-down of a given well, 1 ft. in diameter and penetrating 50 ft. of water-bearing strata, is 32.9 ft. Under otherwise similar conditions the draw-down of a well of six times the diameter penetrating the same depth of strata is 25.3 ft. (77% as great) while that of a well of the same diameter and penetrating twice the depth of water-bearing strata is only 17.0 ft. (52% as great). In wells of Type *C* it is obvious that as long as the cavity at the bottom of the well has a greater diameter than that of the well-tube the latter will not affect the draw-down curve. In ordinary materials

this means that the diameter of the well is not a factor unless it is very large. As shown in Fig. 8, open-bottom wells, even if of large diameter, will have a relatively great draw-down. These computations indicate that under uniform conditions the capacity of wells may be much more readily increased by increasing the depth of water-bearing strata penetrated than by increasing the diameter.

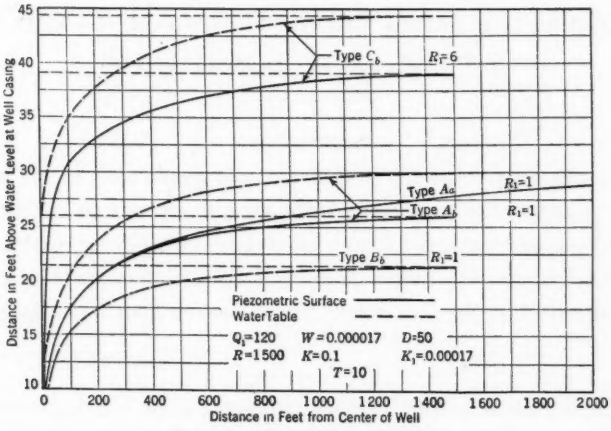

FIG. 8.—DRAW-DOWN CURVES.

When consideration is given to the fact that the strata lie normally in horizontal beds, the desirability of penetrating as deeply as possible into the zone of saturation, as long as the various water-bearing strata are interconnected, is emphasized.

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance of O. V. P. Stout, M. Am. Soc. C. E., in the preparation of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

GRAVITY DAMS ARCHED DOWN STREAM

By B. J. Lambert,[1] M. Am. Soc. C. E.

Synopsis

When a canyon site seems best adapted to a gravity type of dam there is some question as to the desirability of arching it in an up-stream direction. The writer advocates the straight gravity dam for certain sites as, for example, the St. Francis Dam, and presents further arguments to support his theory that in some situations a dam curved down stream may be preferred to the other two types.

The writer has not seen, in discussions on high, curved, gravity dams in canyons, any reference to the relation of the curvature of the dam to the slopes of the side walls. He has been interested in this problem for a number of years, particularly since the failure of the St. Francis Dam in March, 1928. Fig. 1 shows a concrete model of this dam built to a scale of $\frac{1}{240}$, or 1 in. = 20 ft., in a trough made to resemble, roughly, the valley at the site. This trough was made of planking as shown.

The model was formed in place. It had a base of about 8.25 in., a height of 10.25 in., and a cross-section similar to that of the St. Francis Dam, with the exception of the heel on the lower 20 ft. The radius of the arch was 500 ft., or, for the model, 25 in. Sections and elevations of a dam illustrating a supplementary feature are given in Fig. 2. This feature is described in another part of this paper.

Assume that the dam acts as a unit, instead of treating each foot in length as a gravity dam. In the model it is readily seen that, if the dam overturns, it will turn about a part of the toe at the deepest section. The toe along the slopes may be under no pressure when the dam starts to overturn, but the sloping ends of the dam are hanging as cantilevers on the central part of the structure. Consider that the curvature of the dam and the slopes of the canyon walls are such that the extreme ends of the dam are in front of a vertical plane passing through the toe of the highest section. Then if the dam begins to overturn the greatest pressure will come first upon these extreme ends, and the remainder of the toe will be under smaller pressures.

In the model as constructed, the toes at the ends are found to be about 3.5 in. (70 ft.) back of the toe at the deepest section. Had the slope of the

Note.—Written discussion on this paper will be closed in **August, 1931**, *Proceedings*.

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canyon walls been somewhat less as shown by the dotted line in Fig. 1(*d*), the ends of the dam could easily have been in front of the toe at the deepest section, in fact, one end was in front of the toe, in the St. Francis Dam.

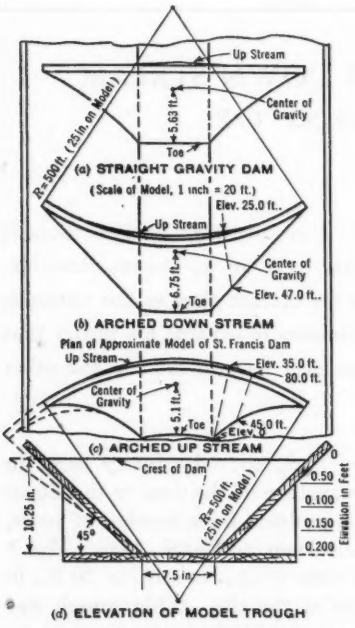

FIG. 1.—MODEL OF THREE TYPES OF GRAVITY DAMS.

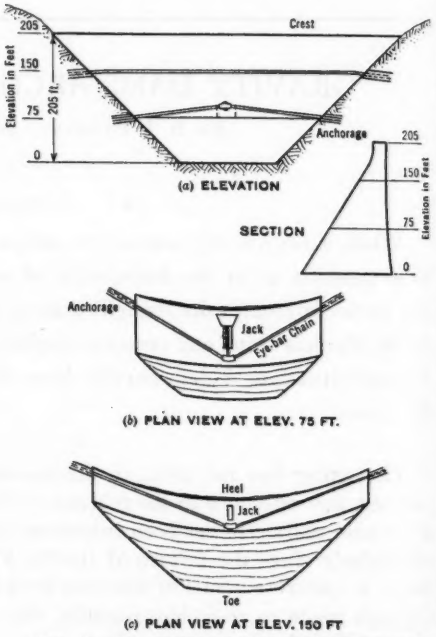

FIG. 2.—DRAWINGS OF MODEL, ILLUSTRATING USE OF EYE-BARS CHAINS.

The effect of curving the dam causes the center of gravity of the dam as a unit to fall nearer the toe. In the model the gravity center is found to be about 5.1 in. from the toe (see Fig. 1(*c*)) while for a small length of dam at its deepest section the center of gravity is about 5.5 in. from the toe.

Likewise, the center of pressure of water acting on the rear of the model dam is found to be about 4.14 in. above the base, and the total pressure to be about 25.7 lb. If full upward pressure is assumed at the heel, varying to zero at the toe, this will be approximately 22 lb. and will be applied slightly inside the middle third of the base. The model dam weighs approximately 48 lb.

Fig. 3 shows a cross-section of the model dam at the middle. If a center of moments is taken at the toe, it will be found that a push, p_1, of $\left(\frac{48 \times 5.1}{10.25} =\right)$ 23.8 lb. is required at the top to overturn the dam, with reservoir empty. By actual test a force of 24 lb. was required. If applied at the center of water pressure, that is, 4.14 in. above the base, the theoretical force, p, required is $\left(\frac{48 \times 5.1}{4.14} =\right)$ 59 lb. The force as actually measured was about 58 lb.

The resultant of the weight of the the dam and the water pressure is found to cut the foundation about 2.9 in. from the toe. Assuming that a section 7.5 in. wide takes all the pressure at the time of incipient overturning, the unit pressure per square inch on the model at the toe is,

$$p = \frac{2\,W}{L}\left(a - \frac{3\,a}{L}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, L is the width of the base, and a, the distance from the toe to the resultant. Then, $p = \frac{2 \times 48}{8.25}\left(2 - \frac{3 \times 2.9}{8.25}\right) = 11.0$ lb., if applied over a length of dam of 1 in., and $\left(\frac{11.0}{7.5} =\right)$ 1.47, if applied over 7.5 in. in length.

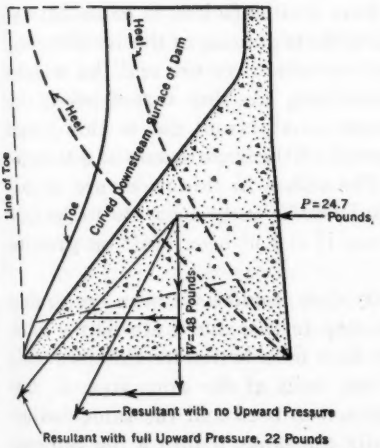

FIG. 3.—SECTION AT THE CENTER, FIG. 1(c), ARCHED UP STREAM.

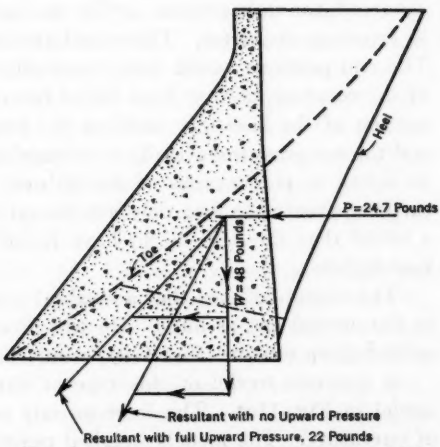

FIG. 4.—SECTION AT THE CENTER, FIG. 1(b), ARCHED DOWN STREAM.

Since the scale of the model is 1 in. = 20 ft. = 240 in., the stress, p, at the toe in the full-sized dam would be $1.47 \times 240 = 350$ lb. per sq. in. on a horizontal section, and about 500 lb. per sq. in. if the actual thrust resultant and a plane normal to it are used for the computation. With full upward pressure at the heel, varying to zero at the toe, the factor of safety against overturning about the toe is small, because the resultant comes within about 1 in. of the toe in the model.

The foregoing remarks indicate that if the overturning of the dam occurred about the full length of the toe, at its deep section (that is, 7.5 in. in the model, or 150 ft. in the original dam), the pressure per square inch on the toe would be only 500 lb., which would not cause failure. If the bottom of the dam along this section is not horizontal, however, but is built in a valley the greatest depth of which occurs over a short distance only, the turning might take place over a much restricted length of toe, and the pressures would increase correspondingly. For example, if in the St. Francis Dam, the toe on a section only 50 ft. long carried the full load at the time of greatest stress, the unit

pressures might then be 1 500 lb. per sq. in., even with no upward pressure considered.

With upward pressure considered and with the resultant cutting the foundation 1 in. from the toe, the unit pressures as determined by Equation (1), are,

$$p = \frac{2 \times 48}{8.25}\left(2 - \frac{3 \times 1}{8.25}\right) = 19 \text{ lb. per sq. in.}$$

If applied over the center section (7.5 in. wide at the toe), this would give more than 2.5 lb. per sq. in. on the model, or $2.5 \times 240 = 600$ lb. per sq. in., on the section horizontal to the thrust, and 850 lb. on the section normal to the thrust.

The section of the St. Francis Dam left standing was only about 90 ft. long. If this represents the length of base on which turning might have occurred, the unit stresses on the toe may have been sufficient to cause failure by crushing at the toe. This may have caused the beginning of the overturning. The end portions would then break off, due to water pressure and the weight of the concrete, as they were lifted from their base. As they turned about the portion of the base left standing the pressures on the front due to this hinge and the toe pressures already mentioned spawled off the large masses of concrete, as shown in photographs of the failure.[3] The values in this paper are necessarily approximate, but they are thought to be sufficiently accurate to justify a belief that the dam might have failed even if it had been built on granite foundations.

The writer believes that a straight gravity dam would have been preferable to the curved one as built. He will go one step farther and state that a dam arched down stream (Fig. 1(*b*)) would have been even better for this location.

A concrete model of this type of dam was built at the same time as the model in Fig. 1(*c*). The same gravity section was used and the same radius of curvature. The models weighed practically the same, within a few ounces, and an analysis of the two volumes seems to show practically equal quantities of materials required for the two conditions. (See Figs. 1 and 4.)

In the model with its back down stream, however, the center of gravity was about 6.75 in. from the toe (see Fig. 1(*b*)). In consequence, the resultant of the total water pressure on the rear and the total weight of the dam cut the base back of the mid-point, and the unit pressures under any of the conditions assumed in the foregoing example were materially reduced.

Theoretically, a force, P, of $\left(\frac{48 \times 6.75}{4.14} =\right)$ 78 lb., is required at the center of pressure to overturn this dam about the toe. In the model, about 80 lb. was required. This, compared with the water pressure of 25.7 lb. on the back of the model, gives a factor of safety of about 3, if upward pressure is neglected.

In the case of the dam with its back up stream, the factor of safety under the same conditions is about 2.3; but with an upward pressure varying from full load, *wh*, at the heel to zero at the toe, the dam with its back up stream

[3] *Proceedings*, Am. Soc. C. E., October, 1929, Papers and Discussions, p. 2157.

has practically no factor of safety, while the one with its back down stream still has a factor of safety of more than 1.5.

It is also to be noted that if a radial section is studied, its base, in the case of a dam arched down stream, slopes up toward the toe; while in the case of the dam the arch of which is up stream, the base slopes down toward the toe. The amount of this slope varies from about 35 ft. at the deepest section to, say, 6 or 8 ft. at the ends.

It is presumed that the water pressure on a gravity dam arched up stream will tend to close any shrinkage cracks, the sections acting as voussoirs of an arch. It might seem as if the dam that was arched down stream would have a greater tendency to open up along shrinkage or construction joints than if it was curved the other way. The writer can scarcely concede even this when it is observed that the entire dam is much more stable, and any radial unit must be tipped up-hill to cause failure. The wings tend to tip against the walls of the canyon when the back of the arch is down stream, instead of tipping into the canyon as is the tendency when arched up stream.

A model of a straight gravity dam was also built (see Fig. 1(*a*)), and was found to be somewhat more stable than the dam arched up stream (Fig. 1(*b*)), and considerably less stable than the one arched down stream (Fig. 1(*c*)). In all cases, the overturning pressures were applied at the center of pressure through a bent hook and a standardized spring balance, using a horizontal pull.

The writer is strongly of the opinion that, before any definite design for a dam in a canyon is accepted, relatively large models of the actual canyon, with its actual foundation profile, and of the proposed dams and footings, should be made and studied. Had this been the practice in the past it is thought that quite likely some of the accepted designs might have been considerably modified, or the footings and abutment conditions materially changed.

It will be observed that the basis of this study is an assumption that, in general, the dam will act as a unit. Construction and shrinkage cracks cannot be eliminated, but the sections will interlock more or less, and certainly will be of large size, as illustrated by the central block of the St. Francis Dam which was left standing after the disaster.

Another suggestion that could be applied to the arched dam with its back down stream is illustrated in Fig. 2. Consider the St. Francis Dam, for example, and assume the curvature reversed. Fig. 2(*b*) shows the dam built up to a height of, say, 75 ft. At this elevation place an eye-bar suspension chain of any cross-section desired, and anchor it at each end into down-sloping tunnels in the canyon walls. After the chain is placed, and the ends are anchored, jacking pressure could be applied to a saddle at its center, and any predetermined initial stress could be maintained while concrete is being placed over the cable. As many cables or chains as desired could thus be placed at such elevations and of such cross-section as would be found most satisfactory. A small percentage of the cost of the dam thus expended in steel (preferably of high tensile strength), would increase the factor of safety both against overturning and sliding.

It seems reasonable to assume that the quantity of concrete could be reduced to such an extent as to pay for the steel and still have a safer dam. It could scarcely be said that these eye-bar cables would act as tensile reinforcement to tie the large blocks of the dam together, because the percentage of steel compared with the quantity of concrete would necessarily be small. However, picture a chain of, say, ten eye-bars, each with a section of 12 by 2 in. (240 sq. in. in all), made of a steel with an ultimate capacity of 100 000 lb. per sq. in., or more. With such a chain, requiring a force of 24 000 000 lb. to break it, one might be justified in expecting that the larger segments of a dam would resist separation for some time.

All this may seem unorthodox, but no harm and, perchance, some benefit may result from a discussion of the proposals herein outlined.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ENGINEERING FOUNDATION COMMITTEE ON ARCH DAM INVESTIGATION

ARCH DAM INVESTIGATION

Discussion

BY EUGENE KALMAN, M. AM. SOC. C. E.

EUGENE KALMAN,[1] M. AM. SOC. C. E. (by letter).[1a]—In Part IV, contributed by Professor Slater, the analysis described is most searching. The writer would like to discuss the more important aspects of Professor Slater's contribution in presenting his ideas as to the stress distribution in shells.

Referring to Section 40, the crack along the central cantilever that developed between the bottom and Elevation 60 may have been caused to a certain extent by initial stresses. Nevertheless, axial tension also must have played its part, because otherwise the extension of the crack along both faces of the dam could not be explained.

From Fig. 85(*a*) it is apparent that, under a 40-ft. head, the radial displacements of the central cantilevers are larger than those of cantilevers 3, 5, and 10 ft., respectively, off center at all elevations. This shows that at least before the crack occurred, the increasing curvature at the center was most likely to form cracks on the down-stream face. However, from a comparison of the curvature conditions at different elevations, it appears that the bending moments were larger at Elevations 40 and 50 than at Elevation 60. In other words, if the cracks had been caused entirely by bending moments they might have been expected to occur at places in the dam rather lower than at Elevation 60.

These facts are illustrated most clearly in Fig. 84(*b*) and Fig. 85(*b*). It is possible to say, of course, that these diagrams represent conditions as they were before the crack occurred, but Fig. 85(*a*) seems to indicate that the

NOTE.—The Report of the Committee of Engineering Foundation on Arch Dam Investigation was published in May, 1928, *Proceedings*, Pt. 3. Discussions of the Report have appeared in *Proceedings*, as follows: November, 1928, by Messrs. Roy M. Green, Harry O. Wood, and R. A. Sutherland; January, 1929, by Messrs. Camillo Guidi, A. Nadal, Nils Buer, and William M. Kinney; April, 1929, by Messrs. B. A. Smith, E. Probst, and Lars R. Jorgensen; May, 1929, by B. F. Jakobsen, M. Am. Soc. C. E.; August, 1929, by Messrs. Roy W. Carlson, and J. L. Savage and Ivan E. Houk; and May, 1930, by Fredrik Vogt, Assoc. M. Am. Soc. C. E.

[1] Visiting Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

[1a] Received by the Secretary, November 2, 1928.

bending moment at Elevation 60 was very slight, even before the crack occurred.

In any case a negative thrust was required before the crack in question could have formed. Basis for this conclusion may be detected easily from a careful study of Figs. 86 to 89, inclusive. Near the abutment, at the higher elevations, considerable radial deflection in a reverse direction is found. This cannot be due to a uniform water pressure; it must be due, rather, to reverse horizontal shear acting radially along the individual arch elements. The magnitude of these deflections indicates the presence of considerable horizontal shear in the region of the haunches. The intensity of this shear may be estimated by bearing in mind the fact that a given load causes a considerably greater deflection at its point of application when applied at the center than when applied in the region of the haunches, which explains the occurrence of a negative thrust in the central section due to a large load acting in the reverse direction near the abutment.

One may object that Figs. 86 to 91, inclusive, were drawn from data obtained in Tests 9 to 13, inclusive, whereas the crack in question had occurred previously in Test 8. It could be claimed that the reverse deflections may have been relatively small if the crack had not occurred. However, a little thought will demonstrate that the presence of the crack permits the loads that act in the central region to cause quite large deformations. Therefore, the extent of the reverse deformations near the ends would be considerably larger compared with the deflections along the center, for the latter are large mainly because of the crack. The effect of the crack is to divide the arch element into two curved horizontal cantilevers entirely free from mutual interaction until the crack closes again.

As to the date of the formation of this crack, Professor Slater states, without proof, that it could scarcely have occurred under a head of water less than 50 ft. Conversely, the writer believes that its formation under a head of water greater than Elevation 50 is not probable. By comparing Figs. 86 and 87 it may be noted that heads of less than 50 ft. seem more damaging in this respect than heads of 60 ft., since in the latter case the negative ordinates of the deflection curve are considerably reduced and the positive deflections are greatly increased; that is, comparatively. Here, deflections are considered as positive when directed toward the center.

The occurrence of this crack is the most significant feature of the Stevenson Creek Arch Dam experiments because it seems to have been caused by the shear. Previously, this shear has been neglected because it has always seemed unimportant, especially in the higher regions where the direct load is negligible. It is extremely interesting to note that the break occurs at a place with which the water does not come into actual contact.

The peculiar shapes of the curves in Fig. 77, representing changes in the widths of the crack at Elevation 60 on the various heads of water, are worthy of note. Evidently, the deflections near the top of the dam depend on two factors, namely, the effect of water load applied directly on the upper portions; and the influence of the deflections of the lower arch elements transmitted to the higher arches through the medium of the vertical cantilevers.

It is easy to see that the deflections of the lower arches will not be appreciable until the water reaches, say, Elevation 25 or 30, because the lower arches are comparatively inflexible. Fig. 3, showing the elementary arches at various elevations, indicates that considerable load is required to cause the lower arches to deflect enough to influence the higher arch elements through the medium of the vertical cantilevers. Conversely, for heads greater than 30 ft., the deflection of the lower arch elements due to the increased load will be much more likely to transfer considerable loads, through the vertical cantilevers, to the upper arch elements which are influenced, in addition, by direct loads.

The occurrence of the vertical crack near the bottom of the dam is also an interesting phenomenon for speculation. The arch slice at Elevation 5 is short and thick. Therefore, it cannot be analyzed by the usual procedure of cutting sections, that is, by the hypothesis of Navier and Bernoulli. An approximate analysis may be made, assuming that the arch is an ordinary fixed beam supporting a uniformly distributed load. Under this condition the bending moment at the ends should be double that at the center. If this is true, the question arises as to why the maximum tension should have been found to occur at the center instead of at the ends.

One answer is indicated, namely, that no telemeter station was established at the very end of this arch. The nearest telemeters are at Stations 7 and 8 (see Fig. (28)). The first problem is to determine whether the position of these telemeters is such as to register larger stresses than those indicated by Telemeter 6 at the center.

Measured by a scale, Telemeters 7 and 8 are found to be 0.36 in. from the center line, as shown in Fig. 28. The length of the beam at Elevation 5 scales 1.0 in. The moment at the center is equal to:

$$M_1 = -\frac{1}{24} l_q^2 = -\frac{1}{24} \times 1.0^2 \times q = -0.0417\ q \ldots\ldots\ldots (151)$$

At Stations 7 and 8 the moment is equal to:

$$M_2 = \frac{1}{12} \times 1.0^2 \times q - 0.5 \times 0.14\ q + \frac{0.14^2}{2} q = 0.0232\ q \ldots\ldots (152)$$

In other words, at each of these locations the moments are less than at the center. Therefore, Telemeters 7 and 8 should not register tensions greater than that indicated by Telemeter 6 at the center. Even this fact, however, does not entirely answer the question as to why the crack should have occurred at the center and not at the ends.

From a purely logical point of view, this may have been due to any one or a combination of the following factors:

1.—Shrinkage of the concrete.
2.—The total change in temperature from the initial value.
3.—Unequal temperature on opposite faces of the dam.
4.—Yielding of the abutments due to the bending moments at the ends.
5.—The existence of a negative or reverse load with its resulting negative thrust.

Factors 1 and 2 do not explain the occurrence of maximum tension at the center instead of at the ends because they cause an almost uniform tension in

all the fibers of the beam. Factor 3 would explain the phenomenon if the decrease in temperature and the consequent increase in tension at the center occurred on the tension side, or, in this case, on the down-stream face. It is not probable, however, that during September 18 and 19, when the water was at Elevation 60 for the first test (see Table 20), the temperature of the up-stream face had exceeded that of the down-stream face to any appreciable extent. The only remaining possible causes are Factors 4 and 5, except that there is a still larger crack at the up-stream face near the abutments. For this reason, an investigation of the conditions on the up-stream face of the dam would be instructive.

The influence of yielding will be treated subsequently. With regard to the existence of reverse load, Professor Slater does not make any statement.

The cantilevers will be stressed considerably at the section of restraint. Because of the compressive stresses in the vertical fibers on the down-stream face, the horizontal fibers will undergo a considerable transverse expansion in such a way as to affect the efficiency of the concrete. In addition, there is a horizontal crack at the very bottom of the dam, which implies a reduction in the height of the section. Thus, the vertical compression is increased even more and the horizontal expansion is correspondingly increased. For example, if the depth of the crack is about 10% of the original thickness the vertical compression will be increased about 25 per cent.

Fig. 126 shows that the horizontal crack at Elevation 30 is at the down-stream face of the dam. The corresponding moment diagram indicates that at this elevation tension is produced on the down-stream face, and, furthermore, that the moment at Elevation 30 is the maximum of all possible moments. It is obvious, therefore, that the crack had to occur at that elevation.

At first, one might expect that the negative moments would cause cracks in the up-stream face; but this is not necessarily true because the sections are correspondingly thicker at areas of large negative moments so that, actually, the unit stresses are not increased in proportion. A construction joint at that elevation might also play some part in causing that crack. It is worth while to observe, however, that at Elevation 35, where there was another construction joint, the crack that occurred was limited in extent to the region nearest the abutments. This phenomenon may be explained by observing the shape of the moment diagram for the cantilever 10 ft. off center, as indicated in Fig. 126, which shows that at Elevation 33 the moments were decidedly greater than at Elevation 35.

The horizontal crack at the bottom of the dam near the up-stream face is important because it was recorded by the telemeter, 3-*U*, when the head on the dam was only 30 ft. This fact is perplexing. In ordinary construction work the selection and preparation of materials are not made with such scientific care as in the case of this dam. Therefore, if a structure built with such perfect care can withstand a head of only 30 ft., what can one expect from structures built for practical use?

There is no doubt that, with the head increasing, the moment in the region of the crack increases also. Other diagrams should be submitted showing the distribution of stresses on the up-stream face. Figs. 94 and 96 are instruc-

tive, but they illustrate conditions only on one face, whereas the conditions at the other face are of equal importance. Only a simultaneous study of both faces can furnish a satisfactory answer.

According to Professor Slater the maximum change in the width of the crack is 0.0014 in., with a value of $E = 3\,600\,000$. This strain in terms of stress is equivalent to a tension of:

$$\frac{\lambda}{l} E = \frac{0.0014}{6.0} 3\,600\,000 = 840 \text{ lb. per sq. in.}$$

It is not prudent to expose concrete structures to so great a strain. What happened after this crack occurred? Obviously, it opened up and extended until it reached a certain limit and thenceforth the remaining part of the section was acting as a kind of hinge. The work that the designers had originally intended the full horizontal section to perform is now done by a reduced section and by the undisturbed arches at the higher elevations which are influenced by means of the cantilevers.

It follows, therefore, that there was no reason for the added thickness at the bottom of the dam. It should be found possible to design the bottom arches with a reduced section that permits them to deform under load as the need arises, without a break. For several years the writer has emphasized the futility and even the danger of making the bottom sections of the arch too thick.[2] It is useless to assign—by prescribing a liberal thickness—to certain elements of the structure a task which these elements cannot perform. The question is not as to whether a beam or arch acting under a given load may or may not fail because of its exaggerated thickness; the question is, may not a structural element fail because it has been made so thick that it is prevented from transmitting part of its own weight by means of its flexibility to adjacent structural elements?

Section 42 is especially important because it contains the first analysis of the physical properties that may be found at the fixed ends of a dam or beam. The writer cannot help expressing his profound disappointment in the choice of the shape and the change in thickness of this dam. The design adopted has an extraordinary disadvantage in that it is not of uniform thickness and is confined to specific characteristics by a perfectly horizontal bottom and vertical canyon walls. Had it been so, it would have been possible to compare the structural elements at different positions. Every difference in the behavior of individual arch elements would then have been due to the difference in their relative positions. In the present case it is not possible to determine the extent to which the differences in the behavior of the various arch or cantilever slices are due to their different positions or to their different geometrical shapes. As the writer conceives it, the aim of the entire Stevenson Creek Dam experiment was to determine how stresses change owing to their positions in different arches or cantilevers. The interpretation of the results is complicated by the introduction of such factors as variable thickness and a variable center angle. Nevertheless, the results obtained are extremely valuable.

[2] *Energia Elettrica*, July, 1927.

There is a remarkable similarity between the two sets of curves in Fig. 80 that deserves closer examination. The thrusts at the abutments acting along the direction of the chord at the two elevations may be determined, and, obviously, the change in length of these chords will be proportional to the forces acting. If the central angle of a fixed arch is denoted by $2\beta_0$, and the thickness, the radius, and the uniform water pressure are denoted, respectively, by the symbols, t, r, and p, then the thrusts at the center section will be given by the formula:

$$H = p\,r\left(1 - \frac{t^2}{6\,\theta\,r^2}\right) \quad \ldots\ldots(153)$$

in which,

$$\theta = \frac{\beta_0}{\sin\beta_0} + \cos\beta_0 - 2\,\frac{\sin\beta_0}{\beta_0} + \frac{t^2}{6\,r^2}\left(\frac{2\,\beta_0}{\sin\beta_0} - \cos\beta_0\right) \quad \ldots\ldots(154)$$

In order to obtain the total thrust at the fixed ends the effect of the water pressure on one-half the arch must also be taken into consideration, as follows:

$$-\int_0^{\beta_0} p\,r\sin\beta\,d\,\beta = p\,r\,(\cos\beta_0 - 1) \quad \ldots\ldots(155)$$

Therefore, the total force acting normal to the rock surface will be:

$$p\,r\left(1 - \frac{t^2}{6\,\theta\,r^2}\right) + p\,r\,(\cos\beta_0 - 1) = p\,r\left(\cos\beta_0 - \frac{t^2}{6\,\theta\,r^2}\right) \quad \ldots(156)$$

Values of β, t, and r from Table 1 are found to be for Elevation 17, $\beta = 14°$, $t = 3.20$, and $r = 99.00$. At Elevation 33, $\beta = 23°\,30'$, $t = 2.00$, and $r = 99.00$. Therefore: $\theta_{17} = 0.002027$ and $\theta_{33} = 0.002991$, and the thrusts at Elevations 17 and 33, respectively, are equal to:

$$r\,p_{17}\,(0.970395 - 0.084210) = 0.89\,r\,p_{17} \quad \ldots\ldots(157)$$

and,

$$r\,p_{33}\,(0.917060 - 0.0222\ldots) = 0.89\,r\,p_{33} \quad \ldots\ldots(158)$$

In other words, the thrusts seem to be proportional to the intensity of the uniform load as they result from Figs. 130 to 133, inclusive. These curves indicate thrusts for various heads of water on the dam as shown in Table 54.

TABLE 54.—EQUIVALENT LOADS CARRIED BY ARCH THRUST

Head of water, in feet	UNIFORM LOAD, IN POUNDS PER SQUARE INCH, AT TWO LEVELS ABOVE THE BOTTOM OF THE DAM	
	17 ft.	33 ft.
60	1 200	1 600
50	1 100	800
40	700	400

The differences shown in Table 54 are not apparent in Fig. 80 which seems to show that displacements of the rock are exactly equal for a head of 40 ft. while for heads of 50 and 60 ft. the displacements at Elevation 17 exceed those

at Elevation 33 by about 16 per cent. Since it is obvious that the displacements must vary almost in proportion to the magnitude of the thrust, Fig. 80 contradicts the evidence given in Figs. 130 to 133, inclusive.

It might be claimed that contrary to the assumptions made, the load acting along the arch slices is not uniform and that this would explain the inconsistency of the curves. However, this explanation is not satisfactory because the part of the load that is not uniformly distributed is exceedingly small compared with the uniform load. For example, observe the ordinates of the curves for horizontal bending at Elevations 17 and 33 in Parts (*a*) and (*b*) of Figs. 130 to 133, inclusive. Under a water head of 50 ft. the non-uniformly distributed part of the load at Elevation 17, 10 ft. from the center of the line of the dam, is shown in Fig. 136 to be appreciable. Certainly, there are sufficient reasons to question the data at this elevation as will be explained subsequently; and even accepting the validity of these data the peculiar contrast is not explained because then the change in thrust at Elevation 33 would be negligible while at Elevation 17 it would be considerable. In the latter case, for example, the thrust would increase in the ratio of 1 150:800 instead of 1 100:800.

Fig. 79 illustrates conditions that appear reasonable as a matter of common sense. Consider the center cantilevers isolated from all adjacent cantilevers. The amount of radial displacement transferred by this cantilever element to the foundation rock is proportional to the water pressure that acts upon the cantilever. Since this pressure increases in a quadratic proportion to the head of water at any point, the radial displacement at the foundation must be represented by a parabolic curve similar to that in Fig. 79.

Referring to Fig. 81 it is evident that the deflection of the rock along the direction of the chord at Elevation 60 will not be appreciable unless the water level is very high. Fig. 77 indicates that at Elevation 60 the crack is not closed and, hence, one cannot speak of an arch thrust at that elevation, until the water head has approached the highest possible value. It seems contradictory that Fig. 81 should indicate even a slight deflection of the rock before the water level has reached Elevation 50, since the crack is still open up to that time. This phenomenon may be explained by the action of the intangible stresses emanating to the region of Elevation 60 from the elevations where there is no crack or where the crack is closed.

In explaining Fig. 82, Professor Slater states that the agreement between displacements observed and those computed by applying Dr. Vogt's formula is satisfactory. In reality, the results obtained by the formulas are more reliable than they appear to be from a comparison of the curves in Fig. 82. In fact, it was necessary for Professor Slater to determine the value of w, in that formula, which is exactly the thrust computed by Equations (153) and (154). The value for w thus obtained did not agree with the deformation observed at Elevation 17. This result does not indicate any error in Dr. Vogt's formulas, which are intended merely to express the proportionality between Δ and w. It shows only, that the thrust computed and the deformation observed were not in agreement. Application of that formula resulted also in carrying the error into Fig. 82. In correcting the diagram for the load acting on the arch slice at

Elevation 17, the thrust will also be corrected and then the coincidence of the two curves in Fig. 82 at that point will occur automatically.

The most important part of Section 32 is the statement that the arches undergo an appreciable lengthening under the effect of a load. This fact implies the creation of a new and additional system of stresses due to the action of a pair of equal and opposite forces applied at the ends of the arches along the direction of the chord. Considering the arch slice at Elevation 17, which has a central angle of about 24° and a thickness of about 3.20 ft., it is obvious that the forces necessary to produce the rib-lengthening found by experiment, must be quite large. As this arch does not differ materially from the shape of an ordinary beam, the unit tensile strength will be practically,

$$\sigma = \frac{0.037 \times 3\,600\,000}{48 \times 12} = 225 \text{ lb. per sq. in.}$$

43.—Deflections Due to Applied Load.—Results involving negative deflections are the most interesting features of this Section. If the deflection curves at the lower elevations (about 20 ft.) are compared to similar curves obtained by Professor C. Guidi, an incomprehensible contrast is noted. (See Fig. 164.) In the report the maximum deflections for the lower elevations are shown to occur near the crown of the arch while in the Guidi experiments the deflection was found to be small at the crown and large at the haunches.

Despite this contrast there is a surprising similarity between the total distribution of loads acting along the arched slices in both cases. Apparently guided by intuition, Professor Guidi found a parabolic distribution for the load decreasing from a maximum at the haunches to about three-fourths of its original value at the crown. Conversely, it is discovered that the non-uniform part of the load as shown in Fig. 124 (Elevation 30) decreases parabolically from the abutments toward the center, and, furthermore, by computing the average equivalent load carried by axial thrust, assisted by Fig. 132, one finds that the maximum intensity of the variable load is approximately one-fifth of the aforementioned equivalent load. Hence, it is shown that the total load does decrease from the haunches toward the center, to four-fifths of the original value, which agrees with the findings of Professor Guidi. This finding of course may be a coincidence, but it seems desirable to call attention to it especially in view of the marked contrast in the corresponding deflection curves.

Professor Slater calls attention to the sharp break at Elevation 30, as shown in Fig. 83. As the result of perfectly logical reasoning, he discards the hypothesis that this sudden break may be due to the presence of the horizontal crack near the bottom. He then makes the statement that there is, however, some relation between the crack and the amount of maximum deflection as indicated by the similarity in the shapes of the curves in Figs. 76 and 83. In the writer's opinion, the shapes of these curves may be explained without admitting any but an incidental connection between them. Attention has been called to the fact that marked deflections cannot occur near the lower parts of the dam because of unusual dimensions, resulting in an extremely large ratio of thickness to length of the arches unless the water level reaches a certain reasonable head. When the water level is high enough the effect of deflection

in the lower arches on the maximum deflection in the higher arches will increase quite rapidly. This will explain the sharp break in the curve at Elevation 30. It is interesting indeed to note that all curves for the deflection of given points at different elevations under varying heads of water will be of the same general form. On the other hand, the more-or-less pronounced width of the horizontal crack at the bottom of the dam is caused mostly by the bending moment acting on the cracked section. The wedge-shaped appearance of the compressed up-stream face suggests that the load influencing the crack might be limited to an analogous wedge-shaped part of that area. If the central angle of this hypothetical wedge-shaped portion at the up-stream face of the dam is denoted by 2β and the head of water is h, then the total bending moment due to the hydrostatic pressure acting on that area is,

$$\int_0^h 2x \tan \beta (h - x) x \, dx = \frac{1}{6} h^4 \tan \beta \ldots\ldots\ldots\ldots (159)$$

In other words, the width of the crack increases as the fourth power of the head, approximately. As a matter of fact, a fairly smooth fourth-degree parabola may be drawn through the circles on Fig. 76.

Thus, it is shown that similarity between the curves of Figs. 76 and 83 is merely accidental. The curves in Fig. 76 would not change much even if the thickness of the arches was changed. On the other hand, if the thick lower arches were made thinner and the higher ones thicker, a radical change would be found in the appearance of the curves in Fig. 83.

45.—Strains Due to Applied Load.—Some of the findings revealed by Figs. 94 to 103, inclusive, are certain to have a revolutionary effect in changing many of the current concepts as to the manner in which arch elements act. In Fig. 94, for example, the areas of intense strain are focused along the haunches at the mid-height. An arch at Elevation 26 indicates the strain to be almost constant from the crown to the abutments where it increases abruptly. At Elevation 50, the change of the strain along the entire length of the arch element is surprisingly uniform. In the latter case the strain was increasing very slowly at first until it reached a maximum not far from the end, at which point it dropped suddenly to a minimum at the abutments. The rate of this final drop is enormous since in a distance of about only 10 ft., the reduction is from 100 to 200 per cent.

This demonstrates the futility of attempting to establish a so-called economic shape for the arch, choosing its axis and its thickness at the different sections so that it may fit better to a uniform load than the circular arch with constant thickness. Fig. 94 indicates that the actual load acting on any specific arch slice may be far from uniform.

Fig. 102 especially, is worthy of detailed study. The most striking fact revealed is that for all horizontal slices the maximum strain occurs along an approximately vertical line about 30 ft. off center.

48.—Effect of Taper in Lower Part of Dam on Determination of Vertical Moments.—This Section outlines the procedure by which Professor Slater corrected an asymmetrical section of a wedge-shaped beam. His assumption that stresses are linearly distributed along the circular segment taken as

section is not applicable to wedge-shaped beams, especially at sections near the restraint. In a rigorous study of some wedge-shaped beams, applying the theory of elasticity, the writer concluded that[3]:

(1) The distribution of stress on normal or inclined sections is not linear.
(2) The maximum compression and maximum tension in the extreme fibers of a section are not equal.
(3) The stress along the neutral axis is not equal to zero.
(4) The values of both the maximum positive and maximum negative stress in the extreme fibers of sections are different from those obtained by the Navier formula that is ordinarily used.

The percentage of error in Conclusion (4) was found to be 9%, 16%, and 66% for beams in which the wedge angles equaled $\tan^{-1}$ 0.5, $\tan^{-1}$ 0.8, and $\tan^{-1}$ 1.0, respectively. These findings serve to emphasize the need of prudence in applying Navier's hypothesis to beams of this shape.

54.—General Outline of Method of Analyzing Test Data.—Equation (22) expresses the fact that the load acting on the horizontal elements can be established into two systems which may be computed separately in terms of radial deflection and axial thrust. This is an excellent method of procedure. The writer, however, would like to suggest an alternative.

Let $f(x)$ express the law that governs the distribution of load along the arch; $F(x, \xi)$ the deflection of the arch at the point, ξ, when a unit load is placed at the point, x; and $Z(x)$, the equation of the deflection curve. Then,

$$Z(x) = \int_{\varphi}^{\phi_0} F(x, \xi) f(\xi)\, d\xi \ldots\ldots\ldots\ldots\ldots\ldots (160)$$

Since $Z(x)$ may be found by measuring the deflection, and $F(x, \xi)$ may be computed, Equation (160) which is called an integral equation, may be solved to determine the load distribution, $f(x)$. This entails considerable work, which is largely compensated due to the fact that only the values of deflection are required. The values of the thrust are not needed. The method has the following advantages:

(*a*) Only one set of measured quantities is required.
(*b*) Details obtained are more uniform.
(*c*) Incidental errors of measurements may be more easily detected, because only one set of measured quantities needs to be revised and analyzed.
(*d*) The number of experimental data required is reduced.

By this method the telemeter measurements would be used only to check the data obtained by the clinometers, thus reversing the sequence: Deflection curve, load, and axial thrust.

K.—Equivalent Load Carried by Direct Thrust.—At least one interesting feature of the curves in Figs. 114 to 117, inclusive, should be discussed. It is noted that the strains registered by telemeters are consistently greater than those obtained by means of clinometers. One might expect that, since the telemeters were nearer the neutral axis, they would register the smaller values. The distance between the two sets of instruments is not appreciable.

[3] Atti Reale Ist., Lomb., Scienze e Lettere, Vol. LXI, Fasc. XVI-XX, 1928.

The explanation of this phenomenon cannot be determined from data in Sections 56 and 57. It is possible that the strains determined from telemeter readings were computed from stresses measured in both horizontal and vertical directions while, on the other hand, the diagrams prepared from the clinometer readings were drawn by considering only changes of curvature in a horizontal direction. If this is true the curves representing clinometer readings were plotted by means of the formula,

$$M = \frac{\delta^2 z}{\delta^2 x} \times \frac{E I}{1 - \mu^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (161)$$

instead of,

$$M = \left(\frac{\delta^2 z}{\delta^2 x} + \mu \frac{\delta^2 z}{\delta^2 y} \right) \frac{E I}{1 - \mu^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (162)$$

In that case the curves representing telemeter data would require some modifications in order to be comparable with the curves of clinometer readings.

The most important finding described in Sections 56 and 57 is that the thrust is almost uniform along the arches. This is explained mostly by the fact that the arches have too small central angles, which choice is incidentally another weak point in this exceptionally important research enterprise.

In practice, arch dams always have relatively large central angles. In this research study they should have been perhaps even wider than usual so that changes in thrust, covering a wider range, could have been observed.

In Fig. 113, one might expect to find that the horizontal stress at Elevation 40 was a mean between that at Elevation 40 and Elevation 50. On the contrary, the illustration seems to show that the contrast between the stress at Elevations 40 and 50 is more pronounced than that between Elevations 30 and 50. Figs. 118 to 121, inclusive—especially Figs. 120 and 121—show that the direct strain increases noticeably after the crack occurred. Commenting on the fact, Professor Slater states that "this does not seem unreasonable, since the ability of the dam to carry load by bending was decreased by the formation of the crack." The writer cannot agree with this statement. There would seem to be no decrease in the ability of the dam to carry load by bending, but rather by compression.

This is illustrated in Figs. 130 and 131 which compare the distribution of loads before and after cracking. These diagrams show that the equivalent load carried by bending changes relatively little after cracking. The vertical load acting directly on the cantilever, however, decreases appreciably at Elevations 10, 15, and 20, exactly where the increase in arch thrust (see Figs. 120 and 121) is most pronounced. This simultaneous decrease of vertical load and increase of arch thrust is especially marked at Elevation 15.

In other words, the increased thrust is accompanied by decreased resistance in the cantilevers rather than by a diminished bending resistance in the arches. Admittedly, the conditions are difficult to analyze. For example, how can the thrust increase and the vertical bending moment decrease simultaneously, as indicated in Figs. 128 and 129? The increase in thrust implies longitudinal contraction which means deflections toward the center, both in arches and cantilevers. These, in turn, involve increase of the vertical bending moments.

Another feature is worthy of note in connection with Figs. 118 and 119. At Elevation 30, and vicinity, with a water head of 60 ft., the thrust is two or more times as great as with a water head of 50 ft. This is a phenomenon relating to the statical conditions in the shell which is absolutely contrary to that assumed under the usual procedure of analyzing arch slices.

Equivalent Load Carried by Horizontal Bending.—As Professor Slater states, Fig. 124 shows that the equivalent load producing bending causes moments to occur even in the center at Elevation 50 despite the fact that a crack is known to exist at this point. It would be valuable to know the kind of instrument used for the measurements in this case and in computing the thrust on which effective section the stresses were distributed.

For the purpose of making a rough check with the data available, consider the arch at Elevation 50 with the water head at Elevation 60. Fig. 118 indicates that the strain at this level is 0.000065 due to thrust. The compressive stress then, is, $\delta = 0.000065 \times 3\,600\,000 = 234$ lb. per sq. in. On the other hand the magnitude of the bending moment (see Fig. 24) is 34 000 ft-lb. which, for a thickness of 2 ft., gives a maximum bending stress of $\delta = \frac{34\,000 \times 6}{2^2 \times 12^2} = 354$ lb. per sq. in. The proportion between these two values does not seem to be satisfactory since, throughout the entire report, the stresses due to bending are consistently reported lower than those due to thrust.

As a check the relation between these two quantities may be expressed in terms of the algebraic quantities, $\frac{p\,r}{t}$ and $\frac{p}{\theta}\left(\frac{\beta_0 - \sin\beta_0}{\beta_0}\right)$, signifying compressive stress and maximum bending stress, respectively. For the arch under consideration, $\beta_0 = 34°$ and $\theta = 0.00562$; from Fig. 118, $p = 700$, and the influence of the equivalent bending load is so small that it may be neglected. Hence, $\frac{p\,r}{t} = \frac{700 \times 99}{2 \times 12^2} = 240$, and the bending stress is equal to $\frac{700 \times 0.054}{0.0056 \times 12^2} = 48$.

The proportion between these two terms is quite different from 233:354 as determined previously in this discussion, and there are not sufficient data presented to explain the difference adequately. Another inconsistency appears in Fig. 120. At Elevation 60, the telemeter registered an average strain of about 0.000020, while, at Elevation 50, the strain registered is only about 0.000013.

M.—Load Carried by Vertical Bending.—According to Figs. 128 and 129, the cantilever loads are larger before than after cracking. On the other hand (see Fig. 85), the deflections of the cantilevers are less before than after cracking. Ordinarily, one might expect that these deflections would vary directly with corresponding variations in applied loads. This paradox is explained by the fact that, actually there are no cantilevers, but beams, fixed at one end and resting for their entire length more or less on the elastic support furnished by the arches. After cracking, the higher arches deflect more, the elastic support

furnished to the cantilevers is no longer so effective, and a relatively smaller load will cause larger deflections in the cantilevers. The resistance of the elastic support of the arches to the deflection of the cantilevers is illustrated by the presence of the negative load at the higher levels. Figs. 128 and 129 indicate that after the crack occurred these negative loads became considerably reduced at the higher elevations.

It is to be regretted that no satisfactory knowledge has been gained concerning the bottom of an arch dam, which is the most vulnerable part of such a structure. This is due partly to the insufficient number of instruments in use, partly to the occurrence of the crack, and partly to the yielding of the foundations.

63.—Results of Tests to Determine Load Carried by Vertical Bending.—This is a most important Section in that it discusses Figs. 134 and 135. The data presented, however, are not sufficient to solve many of the questions that arise. For example, the telemeters at Stations 7 and 8, and 10 and 11, respectively, are symmetrical about the center line, yet the strains indicated differ widely. At Station 10 no bending is indicated and strains are due entirely to thrust, while at Station 9 the moments are greatest. Even along the center line, at Stations 6, 9, and 12 (Elevations 4, 10, and 20, respectively), the irregularities and differences shown by the curves are perplexing. Especially striking is the fact that while the curves in Fig. 134(*a*) and Fig. 134(*g*) have a vague similarity, the curve in Fig. 134(*d*) for an intermediate elevation is entirely different.

Table 55 was compiled from Fig. 134. Item No. 4 seems to indicate that the distribution of bending moments along the cantilever does not agree with commonly accepted ideas. Even taking into consideration the increased thickness of the section at Elevation 4 compared with that at Elevation 10 it seems impossible that strains at the two levels should vary in the ratio of 8:28. The presence of direct tensile strains at Elevation 20 is also difficult to explain.

TABLE 55.—Strain Conditions for a Water Head of 40 Feet, in Outer Cantilevers

Item No.	Description	Observed Vertical Strains, in Millionths		
		Elevation 4	Elevation 10	Elevation 20
1	Maximum strain at the up-stream face	+8	−16	−8
2	Maximum strain at the down-stream face	+22	+20	−22
3	Strain due to direct stresses	+16	+12	−7
4	Strain due to bending	±8	±28	+15

In attempting to explain the presence of compressive strains at Elevations 4 and 10, one would think of the effect of the weight. Thus, it will be best, first, to determine the approximate pressures derived from the weight of the dam. The specific weight of concrete is about 0.08 lb. per cu. in. The compression due to this dead load at Elevations 4 and 10 will be $0.08 \times 56 \times 12 = 54$

and $0.08 \times 50 \times 12 = 48$ lb. per sq. in., respectively. Conversely, the compressive stresses necessary to generate the average strain of 0.000016 and 0.000012, as indicated on Fig. 134(*a*) and Fig. 134(*d*), will be, $\frac{16 \times 3\,600\,000}{1\,000\,000} = 57$ and $\frac{12 \times 3\,600\,000}{1\,000\,000} = 48$, respectively. It will be seen that these values agree with the compressive stresses computed from the dead weight of the dam.

It should not be forgotten, of course, that with a head of 40 ft., Fig. 134(*a*) indicates compressive strains that are relatively small, while for other heads the conditions are shown to change rather abruptly. In any case, the strains due to direct stresses appear to be of about the same magnitude as those caused by the weight of the dam.

In Fig. 135, Professor Slater emphasized the fact that plane sections do not remain plane after bending. The conditions indicated by the curves in this diagram are considered by Professor Slater to be the result of two causes: (1) The presence of considerable direct stress; or (2) the presence of a crack at the construction joint at Elevation 9.5. He suspects the latter as the true cause. The strain at Elevation 10 as registered by clinometer and plotted in Fig. 126 supports this belief. This latter theory explains entirely the curves in Fig. 135. In fact, between the limit of about Elevation 0.05 and 1.00 ft. above and below, the maximum stresses and strains will occur in the extreme fiber of the secondary cantilever. Furthermore, plane sections will not remain plane because of the existence of a dead strip outside the limit of the theoretical area known as "secondary" cantilevers.

The writer admits the possibility of the presence of a crack; but sufficient other causes may be found which at that elevation would explain the variations from the plane sections indicated in Fig. 135. The intense strain indicated in Fig. 126 may be explained at least partly by the presence of these other influences. Suppose, for example, that the sections become warped because of yielding. In that case, the strains registered on the clinometer will not be the actual warp because this instrument measures the angle between the chord, 5–10 and 10–15, which may furnish a considerably larger curvature than the actual one at Elevation 10.

Sections 64 to 67, Inclusive.—The writer cannot accept as logical the extraordinary decrease in vertical load at Elevation 5 under a water head of 50 ft. (Fig. 133.) Furthermore, why should there be such an enormous increase in thrust? The diagrams for the corresponding loads at other heads are quite regular and seem quite logical and there is no particular reason for the marked irregularity in the case of a water head of 50 ft. The feature of these diagrams that is most incomprehensible is, however, the change in the magnitude of the equivalent load of bending and thrust from water heads of 50 to 60 ft.

A comparison of cantilever loads in Figs. 132 and 133 shows that they are larger for heads of 60 ft. than for heads of 50 ft. at all elevations. It naturally follows, therefore, that there will be larger radial deflection under a head of 60 ft. at all elevations. On the other hand, these radial deflections are common

attributes of both cantilevers and arches. At Elevation 5, for example, the arch deflects appreciably more under a head of 60 ft. than of 50 ft. The load is assumed to be uniform at that elevation and it follows, therefore, that the uniform load causing larger deflection would naturally be greater with a 60-ft. head than with a 50-ft. head. In Figs. 132 and 133, exactly the opposite is indicated.

It may be possible that the vertical crack near the bottom occurred while the water head was being changed from 60 ft. to 50 ft. between experiments. This would result, of course, in larger deflections under smaller loads for a water head of 60 ft. Fig. 133 indicates that there was almost no horizontal bending at any elevation except Elevation 5 where the load amounted to 1 700 lb. per sq. in. This is equal to 50% of the total water pressure at an elevation where one would naturally not expect an appreciable horizontal load. Comparing this curve with that for a head of 40 ft. at Elevation 5, it will be found that there is no bending load present. Everything considered, there seems to be no reason for that sudden increase from zero to 1 700 lb. per sq. in. at Elevation 5.

The occurrence of the vertical crack might be offered as an explanation but that would have occurred, then, between heads of 40 and 50 ft. and not between heads of 50 and 60 ft., as stated in the foregoing remarks. Furthermore, this crack would not have explained why there is no bending load at Elevation 5 in both Figs. 130 and 131; that is, at least under a head of 40 ft. the occurrence of the crack did not cause any change in the intensity of the bending load at any elevation.

These problems are very difficult to solve because Professor Slater has not stated clearly just when the crack occurred nor the manner in which the data were prepared for the plotting of the separate curves. Furthermore, these fail to explain the manner in which the total horizontal load in Table 23 is divided into equivalent loads of bending and of thrust.

As stated previously, the writer is of the opinion that the crack at Elevation 60 occurred if not completely, at least partly, because of the negative load in the haunch region. Professor Slater stated that strains due to bending moments are considerable, and he supports this statement by referring to the vertical crack at the top. The writer, however, cannot accept this conclusion. Professor Slater states that the crack could not have occurred under a head of less than 50 ft. On the other hand, Fig. 87 demonstrates that under a water head of 50 ft. there was considerable reverse load with a corresponding occurrence of axial tension. Of course, on the basis of Fig. 87 alone one might be skeptical as to the intensity of these negative loads as compared with the intensity of the positive loads. However, as shown in the part of Fig. 125 relating to the equivalent bending load at Elevation 60, the loading had a decidedly greater tendency to generate axial tension before the crack occurred than afterward. Hence, it may be expected that the diagram of loading at Elevation 50 represents conditions that are considerably less likely to generate axial tension than those before the crack. The writer wishes to express his admiration of this monumental contribution by Professor Slater.

Part VIII.—The analysis of the structural actions of the Stevenson Creek Arch Dam as presented by Professor Westergaard is very thought-provoking. The writer would like to contribute to the theoretical treatment of shells by suggesting a method which, it seems to him, may be more readily grasped by the average engineer. A brief hint and the general principles of this method have been included in two previous papers by the writer.[4]

Cut the dam into a certain number of isolated arches or cantilevers substituting the statical effect of the removed parts by a system of forces, in addition to the water pressure. This system of forces should represent the actual conditions that exist on the hypothetical surfaces or sections of the arches and cantilevers, while they are parts of the dam as a whole. Of course, the exact distribution of these forces is not known, but by the use of a trial method (by finding certain groups of systems of forces containing a certain number of parameters), it is possible, by the principle of least work, to select the system of forces which will represent the most accurate approximation of the actual stress distribution.

It is an essential condition of the problem that the choice of forces shall be such as to approximate within reasonable limits the actual stresses as they exist in the structure. In other words, this method consists essentially of two distinct steps: First, the selection of a number of convenient correctional force systems; and, second, the determination of the most correct of those systems by means of the principle of least work. The second step is extremely simple. The arch and cantilever slices being acted upon by the water pressure and by the supposed correctional forces, the stresses caused by these applied loads can be computed by means of the simple and well-known formulas and methods. When the stresses at all points are known, they may be substituted into the expression representing the work of deformation, W, as follows:

$$W = \frac{1}{2G}\int\int\int\left[\frac{1}{2}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{1}{2(m+1)}(\sigma_x + \sigma_y + \sigma_z)^2 + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right] dx\,dy\,dz \quad \ldots\ldots\ldots\ldots\ldots\ldots (163)$$

This equation may then be reduced and solved so as to determine the value of that parameter which will make the integral a minimum. There are a number of such parameters and the problem is to determine the theories of systems which seem to represent correctly the actual distribution of stresses acting in the shell along the imaginary cut faces between the single arch and cantilever slices.

First of all, it will be necessary to analyze stresses in the shell. Fig. 182(*a*) represents an arch slice. The forces acting are: The water pressure, w, vertical pressures and bending moments, horizontal shear, acting in a tangential direction, and horizontal shear, τ_{yr}, acting radially.

The vertical pressure, which of course is not shown on Fig. 182(*a*), may be considered an average uniform load caused by the weight of the dam acting on the section in question. Except in extremely high dams, this class of stress

[4] *Energia Elettrica*, July, 1927; and Atti Reale Ist., Lomb., Scienze e Lettere, Vol. LXII, Fasc. VI-X, 1929.

is generally quite small. There is, however, another aspect of this situation to be considered. The weight acts in a vertical direction and thereby causes the dam to be compressed vertically. Consequently, there will be expansion of the arch elements in any horizontal direction due to this vertical load: that is, any arch slice has the tendency to undergo a certain elastic lengthening, and this will induce additional stresses in any statically indeterminate structure.

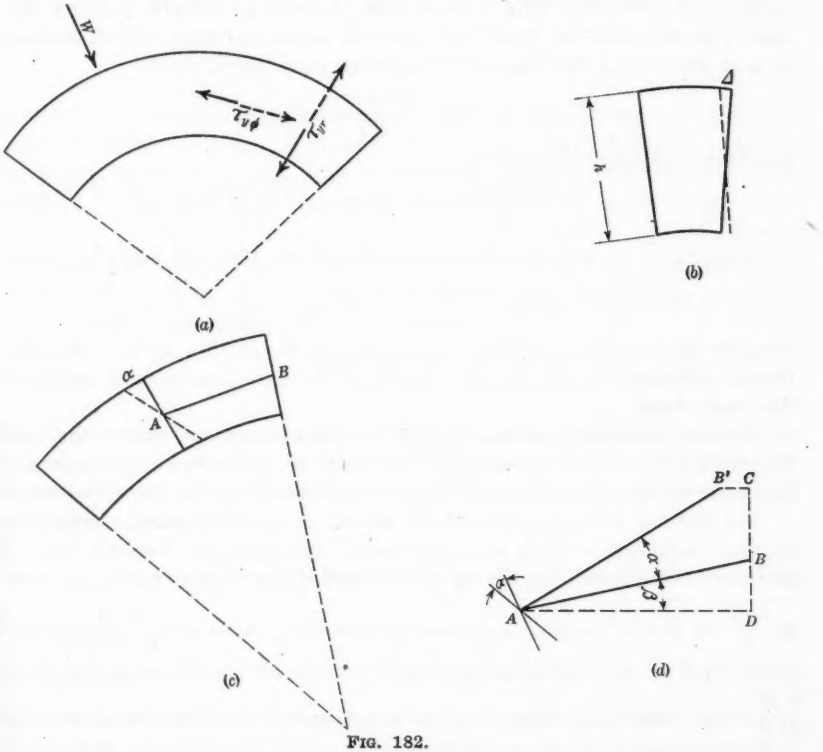

FIG. 182.

It will be instructive to get some idea as to the actual stresses involved in this rib-lengthening in the case of arches fixed at both ends. Consider an arch slice at a distance of y ft. below the crest of a dam. The weight of the column of concrete resting on any square inch of the cut surface is about 0.90 lb. This unit load is uniformly distributed along the entire section on every 1-in. square in this section. The vertical contraction will be $\frac{0.90\ y}{E}$, and the horizontal expansion will be $\frac{0.90\ \mu\ y}{E}$, due to the dead weight of the concrete above the section.

The elongation of a longitudinal element, $r\,d\,\phi$, will be equal to $\frac{0.90\ \mu\ y\ r\ d\ \phi}{E}$, in other words, in a circular segment with a central angle equal to $2\ \phi_0$, there will be a rib-lengthening of,

$$\int_{-\phi_0}^{+\phi_0} \frac{0.90\ \mu\ y}{E}\ r\cos\phi\ d\ \phi = 1.8\ \frac{\mu\ r\ y}{E}\sin\phi_0 \ldots\ldots\ldots\ldots (164)$$

The abutment reactions to this load may be computed by determining the magnitude of the forces necessary to return the ends of the free arch to their original undeflected positions, thus causing a rib-shortening of the same magnitude as the rib-lengthening.

In order to compare these reactions with those produced by the water load it will be necessary to determine the rib-lengthening caused by a uniform pressure of 0.43 y lb. This is the weight of water at a depth, y, below the surface. In this case the total ring stress is equal to $0.43yr$, which corresponds to a shortening in the elementary segment, $r\,d\,\phi$, equal to,

$$\Delta = \frac{0.43\,y\,r}{E\,t}\,r\,d\,\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (165)$$

and the total rib-shortening is,

$$\int_{-\phi_0}^{+\phi_0} \frac{0.43\,y\,r}{E\,t}\,r\cos\phi\,d\phi = \frac{0.86\,r^2\,y}{E\,t}\sin\phi_0 \ldots\ldots\ldots\ldots\ldots (166)$$

The ratio of the two rib-shortenings, Equation (164) and Equation (166), is:

$$\frac{1.8\,\mu\,r\,y}{E}\sin\phi_0 : \frac{0.86\,r^2\,y}{E\,t}\sin\phi_0 = \frac{1.8\,\mu\,t}{0.86\,r} = \frac{t}{4\,r}$$

Because the magnitude of this ratio is negligible in thin arches, the effect of lateral deformation, due to the weight of the dam, can also be neglected in thin arch dams.

Vertical bending moments, acting in radial planes, are due to the bending in cantilevers. Their magnitude can only be estimated in a general way because it is not yet known how they are distributed on the surface of the shell.

The vertical bending moment, M, acting in a radial plane, exerts stresses in that cantilever, which are distributed according to Navier's law. The extreme fiber stresses on the up-stream and down-stream faces are equal to $\pm\frac{6\,M}{t^2}$, in which, the tensile stresses are positive. A stress of $\frac{6\,M}{t^2}$, acting on a square inch of the horizontal arch slice, causes an elastic elongation equal to $\frac{6\,M}{E\,t^2}$ in a vertical direction, and in a horizontal direction, the shortening is equal to $\frac{6\,\mu\,M}{E\,t^2}$. Radial contraction does not alter the stress conditions in the arch, as previously stated.

Longitudinal contractions and elongations involve stresses in a statically indeterminate arch with fixed ends. In order to compute them, consider one-half an arch segment as a statically determinate ring segment. Consider one element of this segment bounded by the extrados and intrados and by the two radii in Fig. 182(b). When the stress distribution is linear the change in length of the extreme fibers will be $\frac{6\,\mu\,M}{E\,t^2}$, while the central fiber, that is, along the neutral axis, will not be deformed. Because the stresses are proportional to their distance from the central fiber (from the neutral axis), the longitudinal deformations will also be proportional to that distance. Hence, the deformed element will be as represented in Fig. 182(b).

Since the stresses are proportional to the distance from the neutral axis, the longitudinal deformation will be likewise proportional to that distance. In this theory, the conditions of the deformed element will be represented by the dotted line in Fig. 182(*c*).

The total deformation of the element includes first of all a rotation equal to,

$$\alpha = \frac{\Delta}{\frac{t}{2}} = \frac{\frac{6\,\mu\,M}{E\,t^2}}{\frac{t}{2}} = \frac{12\,\mu\,M}{E\,t^3} \quad \ldots\ldots\ldots\ldots (167)$$

This rotation, in turn, causes a certain horizontal displacement of the end section. It can easily be seen from Fig. 182(*c*) that a small rotation, α, at the point, *A*, will cause the point, *B*, to be deflected to a new position, *B'*. (See Fig. 182(*d*).) The horizontal component of this deflection is equal to,

$$B'\,C = B\,B' \sin\beta = A\,B\,\alpha \sin\beta = \alpha\,B\,D \quad \ldots\ldots\ldots\ldots (168)$$

The value of α from Equation (167) is substituted in Equation (168). Furthermore, by geometry, $B\,D = r - r\cos\phi$. Then the horizontal deflection of the end section due to the rotation at *A* will be equal to,

$$B'\,C = \frac{12\,\mu\,M\,r}{E\,t^3}\,(1 - \cos\phi) \quad \ldots\ldots\ldots\ldots\ldots\ldots (169)$$

The total rotation and the horizontal deflections are obtained by integrating all the moments acting on the elementary segments of the arch. Denoting these values by δ_m and δ_M, respectively, they may be determined by means of the following equations:

$$\delta_m = \int_0^{\phi_0} \frac{12\,\mu\,M}{E\,t^3}\,r\,d\phi = \frac{12\,\mu\,r}{E\,t^3}\int_0^{\phi_0} M\,(\phi)\,d\phi \quad \ldots\ldots\ldots (170)$$

and,

$$\delta_H = \int_0^{\phi_0} \frac{12\,\mu\,M\,r}{E\,t^3}\,(1 - \cos\phi)\,d\phi = \frac{12\,\mu\,r}{E\,t^3}\int_0^{\phi_0} M\,(\phi)\,(1 - \cos\phi)\,d\phi. \quad (171)$$

Of course, these equations cannot be properly reduced until the value of *M* is determined as a function of ϕ. Professor Slater states that the values of *M* are small except at the bottom of the dam. It is evident that in the region near the abutments there is little or no bending in the cantilever, and, therefore, *M* must be equal to zero. At Elevation 5, along the center line, when ϕ equals zero, the value of *M* is equal to 250 000 lb. per sq. ft. per ft. of width, according to Fig. 176.

In order to have some basis for a comparative analysis, *M* will be assumed to be distributed according to the cosine law. The arch will be assumed as a part of a cylindrical shell with a central angle of 120° and $\phi_0 = 60$ degrees. Then the law of distribution for *M* will be:

$$M\,(\phi) = 250\,000 \cos\frac{3}{2}\,\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (172)$$

It is easily seen that this formula gives values of 250 000 and 0, when $\phi = 0$ and 60°, respectively, as assumed. Substituting $M\,(\phi)$ from Equation (172)

into Equations (170) and (171), the following values will result for the rotation and the horizontal displacement of the end section:

$$\delta_m = \frac{12\,\mu\,r}{E\,t^3}\int_0^{\phi_0} 250\,000\cos\frac{3}{2}\,\phi\,d\,\phi = \frac{12\,\mu\,r}{E\,t^3}\,250\,000\,\frac{2}{3}\sin\phi_1 \ldots (173)$$

and,

$$\delta_H = \frac{12\,\mu\,r}{E\,t^3}\int_0^{\phi_0} 250\,000\cos\frac{3}{2}\,\phi\,(1-\cos\phi)\,d\,\phi$$

$$= \frac{12\,\mu\,r}{E\,t^3}\,250\,000\left(\frac{2}{3}\sin\frac{3}{2}\,\phi_0 - \sin\frac{\phi_0}{2} - \frac{1}{5}\sin\frac{5\,\phi_0}{2}\right) \ldots\ldots (174)$$

Equations (170) and (171) may be reduced by substituting $\phi_0 = 60$, $\cos\phi_0 = \frac{1}{2}$ and $\sin\phi_0 = \frac{\sqrt{3}}{2}$. The results in each case are as follows:

$$\delta_M = 1\,730\,000\,\frac{\mu\,r}{E\,t^3} \ldots\ldots\ldots\ldots\ldots\ldots (175)$$

and,

$$\delta_H = 200\,000\,\frac{\mu\,r}{E\,t^3} \ldots\ldots\ldots\ldots\ldots\ldots (176)$$

The next thing to consider is the effect of the water pressure on this arch slice. The height of the water column is 55 ft. and, therefore, the water pressure will be $55 \times 0.43 = 23.6$ lb. per sq. in. As stated previously, this water pressure will create a ring pressure equal to $23.6r$ and a total rib-shortening of,

$$\delta_W = \frac{23.6\,r^2}{E\,t}\sin\phi_1 = 20.4\,\frac{r^2}{E\,t} \ldots\ldots\ldots\ldots\ldots\ldots (177)$$

By substituting the necessary values, the ratio between Equations (177) and (176) is found to be equal to 3. For purposes of this discussion, the writer is not interested in the complete computation of the actual stresses. He desires merely to compare the relative magnitudes. It is easily seen from the foregoing analysis that the horizontal displacements of the statically determinate half arch under the action of water pressure and of transverse bending moment being commensurable, the stresses generated in the two cases will be of the same order of magnitude. In other words, the effect of the vertical bending moment may not be ignored, at least not at the lower elevation.

There are evidences that the shears lying in the plane of the cut surfaces and acting in a tangential direction must be comparatively small. Professor Slater does not mention them at all. Consider, for example, two horizontal slices in the shell corresponding to the depths, y and $y + d\,y$. The tangential and radial shears occurring between them at any point are to some extent proportional to the relative tangential and radial displacements of the slices in those points. On the other hand, it is easy to see that the flexibility of the arches as well as their relative deflection is incomparably larger in a radial direction than in a tangential direction. This accounts for the small value of the tangential shear as compared with the radial shear.

Next, consider a cantilever slice of elementary thickness, $r\,d\,\phi$. From practical experience it has been found that such an element of the shell undergoes

considerably larger deflection in a horizontally radial direction than in a horizontally tangential direction and this in spite of the fact that its rigidity and its resistance against radial deflections are very great, while its rigidity against tangential deflection is practically zero. Hence, the bending load acting radially on a cantilever slice must be incomparably greater than that acting in a tangential direction. On the other hand, the shares are proportional to the first derivatives of the corresponding load. This accounts for the fact that tangential shear is negligible as compared with radial shear. Shearing moments play an important rôle in the theory of plane slabs, in which the maximum positive and negative tangential shears are of a greater order of magnitude than the shears acting parallel to the load.

This importance in the case of slabs is due to the fact that bending moments change from one slab slice to the other. Let Fig. 183 represent two faces of a slab element, one behind the other. Both are subjected to the action of bending moments which are more or less different. Consequently, the longitudinal displacements of the two faces will be different, the greatest difference occurring along the upper and lower edges of the slab. Hence, shearing stresses of a greater or less intensity will arise, the maximum being at the edges. Their resultant is equal to zero and they form a shearing moment, or twisting moment, which Professor Westergaard designates, M_{xy}. In the case of the cylindrical shell under discussion the moments are of secondary importance compared with the axial thrust. Conversely, in the slabs subject to bending, axial thrust is negligible. In short, according to Professor Slater, the amount of these shearing moments seems to be relatively unimportant.

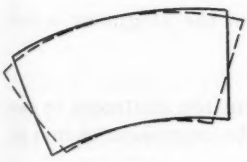

FIG. 183.

FIG. 184.

In conclusion, the following comment concerning the stresses may be made: (1) Normal stresses acting horizontally in a radial direction are very small. They are equal to the water pressure on the extrados and to zero at the intrados varying according to a straight-line diagram between the two faces; (2) shears acting perpendicular to these normal stresses may be neglected; (3) the value of normal stresses acting horizontally in a tangential direction and shears acting perpendicularly to them may be obtained by an analysis of the arch slices: (4) normal stresses acting in a vertical direction and shears acting perpendicularly to them may be obtained by an analysis of the cantilever slices; (5) therefore, all the stresses that are included in the integral of work, Equation (167), can be evaluated and substituted therein. The integral itself may thereafter be computed if the load system acting on the arch and cantilever slices is known.

Consider the conditions for equilibrium in a horizontal radial direction for the set of stresses acting on a differential block, as shown in Fig. 155. The

radial shear acting on the arch slices is $\frac{\delta Q_y}{\delta y}$. From Equation (33) and Fig. 155, it is apparent that the total load acting on the cantilever slices will be equal to,

$$W - \frac{P_x}{r} + \frac{\delta Q_x}{\delta x} = -\frac{\delta Q_y}{\delta y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (178)$$

Assume that the radial shear is the principal unknown quantity and substitute $-f$ for the quantity, $\frac{\delta Q_y}{\delta y}$. Then, the arches are acted upon by the load, $w - f$, as are the cantilevers, by the load, f. If the value of f is known, the stresses can be computed in all places. Since the arches are acted upon by the load, $w - f$, in which, w is the water pressure, the factor, f, may be considered the correcting increment of a load on the arches.

Selections of the Proper Values of f.—Obviously, for $\phi = \phi_0$, f must be equal to zero in a shell limited by two vertical generatrices of a cylinder, because the lateral cantilevers do not undergo any deformation and hence no load can act on them. Besides f must be some even function of ϕ, owing to the symmetry of the system. Furthermore, the load on the arches decreases so that at the center it is about 25% of its value at the abutment. All this evidence points to the fact that f may be assumed equal to $\frac{w}{4} \cos \frac{\pi}{2\,\phi_0} \phi$. For cantilever loads at the abutments, that is, for $\phi = \phi_0$, this expression becomes zero; otherwise this equation is an even function and the arch load, $w - f = w - \frac{w}{4} \cos \frac{\pi}{2\,\phi_0} \phi$, decreases at the rate of $\frac{I}{4}$ from the abutment where $\phi = \phi_0$, to the center where $\phi = 0$.

Evidently, the rate at which the load decreases from the abutment to the center cannot be the same for all arches. Therefore, the expression should be changed to $f = a \cos \frac{\pi}{2\,\phi_0} \phi$, in which, a is a quantity to be determined.

Furthermore, it is obvious that the quantity, $\cos \frac{\pi}{2\,\phi_0} \phi$, may not represent the correction factor, f, in a satisfactory way. While retaining the maximum value at the center and the value of zero at the abutments, some slightly different form of the cosine function may be considered desirable. This may be accomplished by adding a complementary quantity, as follows:

$$f = a \cos \frac{\pi}{2\,\phi_0} \phi + \sigma \cos 3 \frac{\pi}{2\,\phi_0} \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (179)$$

Equation (179) illustrates the fact that the load acting upon the central cantilever, where $\phi = 0$, will be equal to $a + b$. This cantilever slice has a deflection curve similar to that reproduced in Fig. 184. One glance at that deflection curve shows that it involves a positive load at the top and a negative load at the bottom. In other words, a load similar to that shown in Fig. 185 must act on that cantilever.

As a trial approximation, the load diagram and Fig. 185 may be considered a linear function of y. It may not be a linear function, however. For this reason and also because there are two quantities, a and b, to be disposed of, a parabolic distribution will be assumed. This may, of course, practically be reduced to a linear distribution if the principle of least work requires such a curve. In Equation (179) if $a = m\,y$ and $b = n\,y^2$,

$$f = m\,y\cos\frac{\pi}{2\,\phi_0}\,\phi + n\,y^2\cos 3\,\frac{\pi}{2\,\phi_0}\,\phi \ldots\ldots\ldots\ldots (180)$$

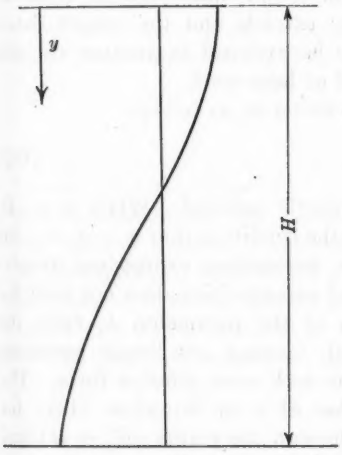

FIG. 185.

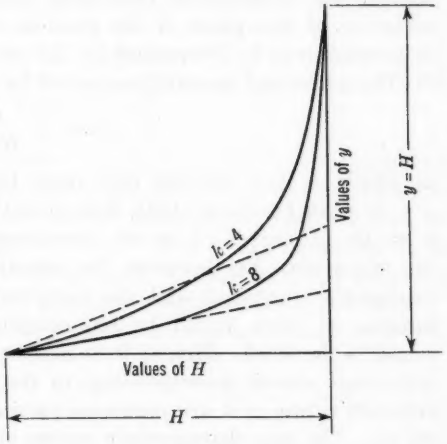

FIG. 186.

The quantity, $w - f$, represents the load acting on the arches and from this fact another expression involving f may be derived. No deformation is possible at the bottom of the dam and, therefore, for any value of ϕ the load, $w - f$, must be equal to zero when $y = H$, the depth of the dam. Since the water pressure, w, may be considered equal to the depth, y (when a satisfactory ratio of proportionality is chosen), the following equation must hold:

$$H - m\,H\cos\frac{\pi}{2\,\phi_0}\,\phi - n\,H^2\cos 3\,\frac{\pi}{2\,\phi_0}\,\phi = 0 \ldots\ldots\ldots (181)$$

However, this relation does not apply for any value of ϕ and, consequently, another form expressing the function of f may be suggested, as follows:

$$f = m\,(H - y)\cos\frac{\pi}{2\,\phi_0}\,\phi + n\,(H - y)^3\cos 3\,\frac{\pi}{2\,\phi_0}\,\phi \ldots\ldots (182)$$

While Equation (180) is a parabola of the second degree, Equation (182) is a parabola of the third degree. The use of Equation (182) may or may not affect the result appreciably, but from the foregoing discussion it is not possible to predict which of the two parabolas might fit the conditions best. Equation (182) seems to be quite satisfactory because it indicates a certain harmony and homogeneity in its terms. By substituting $y = H$ for $w - f$, its value will reduce to H, which is not equal to zero as it should be; however, H in

this case is a constant and, therefore, comparatively easy to manipulate. The result is that Equation (182) is a decidedly improved form which, in turn, may be further simplified by the addition of one more quantity which will furnish $-H$ for $y = H$. Many expressions of this form may be found, of course. In selecting the simplest form it should be remembered that the arch dam recovers its capacity to deform rapidly as the distance from the bottom arches increases. The significance of this fact is that the influence of the third quantity to be added to Equation (182) should be more or less localized near the bottom of the dam. It is not possible to determine how far from the bottom the influence of this third quantity extends and the proper determination of this phase of the problem must be assigned to another variable or parameter to be determined by the method of least work.

The additional quantity suggested by the writer is, as follows:

$$z = \frac{y^k}{H^{k-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (183)$$

in which, k is a variable that must be properly selected. When $y = H$, $z = -H$ in Equation (183), thus satisfying the condition that $w - f = 0$ for $y = H$. However, it is not convenient to manipulate expressions involving exponents. Furthermore, the principle of superposition does not hold for exponential functions and the computation of the parameter, k, from the integral of work would be too complicated, because non-linear equations would be involved. Hence, it is necessary to seek some simpler form. The individual curves corresponding to the value of z in Equation (183) for different values of k are parabolas passing through the points $(H, -H)$ and $(0, 0)$. The two characteristic curves shown in Fig. 186 illustrate diagrammatically that they are more or less straight except for the higher values of H. From this point of view, the following formula may be accepted for use instead of Equation (183):

$$z = H - c\,(H - y) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (184)$$

In Equation (184), the coefficient, c, is the inclination of the segment with respect to the z-axis. This expression is assumed to hold only for the positive segment of the curves shown in Fig. 186; that is, between the values of $y = \frac{c-1}{c} H$ to $y = H$. Between the values, $y = 0$ to $y = \frac{c-1}{c} H$, the value of z is assumed as equal to zero. This assumption is permissible since the additional load is not expected to have any effect except in the lower regions. The addition of Equation (184) to Equation (179) results in a final form, as follows:

$$f(y, \phi) = m\,(H - y) \cos \frac{\pi}{2\,\phi_0}\,\phi + n\,(H - y)^3 \cos 3\,\frac{\pi}{2\,\phi_0}\,\phi + H - c\,(H - y) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (185)$$

In Equation (185), the condition is fulfilled that,

$$w\,(H) - f\,(H, \phi) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (186)$$

This relation holds for any value of ϕ, but because of all the corrections in the function, $f\,(y, \phi)$, from the original, Equation (185) no longer satisfies the

condition required for the cantilevers at the abutments; namely, that $f(y, \phi_0)$ shall equal zero. Substituting $\phi = \phi_0$ in Equation (185), results in the following relation:

$$f(y, \phi_0) = H - c(H - y) \quad \ldots\ldots\ldots\ldots\ldots\ldots (187)$$

which is not equal to zero.

In order to avoid this difficulty, another function may be used similar to Equation (184), in that its value should equal H for any value of ϕ. It will differ from Equation (184) in that its inclination with respect to the y-axis should decrease toward the abutments. At the abutments, the inclination should equal zero, and the curve should become a horizontal line. One example of such a function is, as follows:

$$H - c\left(\frac{H - y}{\cos \frac{\pi}{2\phi_0}\phi}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (188)$$

It seems that Equation (188) would prove troublesome since it becomes infinite at the abutments for any value of y. This is not the case, however, because for any value of y and ϕ which makes the second term greater than the first, the entire expression would have a negative value. This means according to the foregoing statements that Equation (188) must be made zero for all sets of values of y and ϕ in order to furnish a negative value for that function.

The final expression for f and $w - f$ may be written as follows:

$$+ (x_1\ \phi) = m(H - y)\cos\frac{\pi}{2\phi_0}\phi + n(H - y)^3 \cos 3\frac{\pi}{2\phi_0}\phi + H - c\frac{H - y}{\cos\frac{\pi}{2\phi_0}\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (189)$$

and,

$$-(w - f) = m(H - y)\cos\frac{\pi}{2\phi_0}\phi + n(H - y)^3\cos 3\frac{\pi}{2\phi_0} + (H - y)\left(1 - \frac{c}{\cos\frac{\pi}{2\phi_0}\phi}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (190)$$

It can be proved easily that Equations (189) and (190) fulfill all the conditions imposed. One might criticize Equation (189), because when substituting $\phi = \phi_0$, the result becomes indeterminate and absurd as follows: $f(y, \phi_0) = H - \infty$, for any value of y. In this case, however, the substitution, $\phi = \phi_0$, is not permitted. At the abutments it will be necessary to compute the values of $f(y, \phi)$ by imposing special conditions for this special case. For example, the last terms of Equation (189) will have the value,

$$H - c\frac{H - y}{\cos\frac{\pi}{2\phi_0}(\phi_0 - \varepsilon)} = H - c\,N(H - y) \quad \ldots\ldots\ldots\ldots (191)$$

in which, N represents an extremely large quantity. The entire expression is positive until $c\,N(H - y) < H$, or, $H - y < \frac{H}{c\,N}$. The significance of this

limitation is that the value of y must be nearly equal to that of H. If this is not the case, then the expression, $H - c\,N\,(H - y)$, becomes negative, with the assigned value, 0, as already explained. Hence, in Equation (189), $f(y, \phi_0) = 0$ holds for every value of y. The writer wishes to emphasize the fact that the expression, $H - c\,\dfrac{H - y}{\cos \dfrac{\pi}{2\,\phi_0}\,\phi}$, represents a special function which is obtained partly by substitution and partly by definition.

When the load acting on the arches and the beams has been chosen, the stresses can be computed both in the arches and in the cantilevers. All these values may be then substituted into the integral of the work of deformation. There are three parameters in the expression that involves the correcting load. The principle of least work supplies the means for determining the most probable values of these three parameters. When their numerical values are determined and substituted in Equations (189) and (190), the correcting load is completely determined and the actual stresses in the single arch and cantilever slices in the shell may be computed.

There is still another point which must be made clear. In the lower parts of the dam it was found that the vertical bending moments, M, will act in radial planes and that because of the lateral expansion and contraction caused by those bending moments, longitudinal deformations will occur in the arches. The magnitude of these deformations will correspond to those accompanying a hypothetical bending moment, $\dfrac{\mu\,M}{E}$, which is assumed to act in a horizontal direction. Corresponding stresses will be of the same order of magnitude as stresses caused in the arch by the action of water pressure; considered jointly their effects can not be neglected. A correction is required to allow for the effect of the movement. The stresses may be computed by various methods, two of which are given herewith.

First, remember, that the load, $f(y, \phi)$, as defined by Equation (189) determines the vertical stresses in the cantilevers and, therefore, in the bending moments. In the foregoing procedure these moments and the horizontal moments, $\dfrac{\mu\,M}{E}$, have been neglected. It is not necessary to neglect them, however, since they may be computed. There is one difficulty, namely, that the expression for the load of the arches, $w - f$, becomes zero when $y = H$. This limiting condition is necessary in order to agree with the condition that the lowest arch which is fixed at the bottom does not deform. If an additional system of moments, $\dfrac{\mu\,M}{E}$, is assumed to act at the lowest arch, this element should undergo certain deformations which are absolutely contrary to the real conditions.

In order to avoid that difficulty, the load system, $w - f$, must be completed by the addition of another load of such a magnitude as to neutralize the action of the bending moments, $\dfrac{\mu\,M}{E}$, on the arch, $y = H$. This, in turn, raises

the new problem of how to determine a system of loading that is certain to generate a given bending moment, $-\frac{\mu M}{E}$, at any section of the arch.

Such problems arise generally when either the load or the deflection is given and the other is to be determined. The two factors are expressed in common terms by means of certain definite integrals, such that when the load is given, the evaluation of the deflection is a problem of simple integration. When the deflection is given, on the other hand, the determination of the value of the load requires the solution of an integral equation as described previously in this discussion. (See Equation (160).) If the bending moment is given corresponding to a certain unknown load the solution will be as follows:

Consider an arch with fixed ends subjected to the action of symmetrical loads, $P\,P$. The magnitude of the reactions, M_0 and H_0, acting at the center section of the arch at its center of gravity is expressed by the following equations:

$$M_0 = P\,r\,\frac{1-\cos(\phi_0-\phi)}{\phi_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots (192)$$

and,

$$H_0 = 2\,\frac{P}{D}\left[\frac{1-\cos(\phi_0-\phi)}{\phi_0} + \left(1+\frac{t^2}{3\,r^2}\right)\frac{\phi_0-\phi}{2\sin\phi_0}\sin\phi - \frac{1}{2}\left(1-\frac{t^2}{6\,r^2}\right)\sin(\phi_0-\phi)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (193)$$

in which, D (when considering the effect of axial thrust and shear), is determined by the equation:

$$D = \frac{\phi_0}{\sin\phi_0} + \cos\phi_0 - \frac{2\sin\phi_0}{\phi_0} + \frac{t^2}{6\,r^2}\left(\frac{2\,\phi_0}{\sin\phi_0} - \cos\phi_0\right) \ldots\ldots (194)$$

For a load system with a continuous distribution, the corresponding reactions may be obtained by substituting $p\,(\phi)\,r\,d\,\phi$ for P, and integrating. Under these conditions the new expressions for the reactions will be:

$$M_0 = \frac{r^2}{\phi_0}\int_0^{\phi_0} p\,(\phi)\,[1-\cos(\phi_0-\phi)]\,d\phi = \int_0^{\phi_0} p\,(\phi)\,R\,(\phi)\,d\phi \ldots (195)$$

and,

$$H_0 = \frac{2\,r}{D}\int_0^{\phi_0}\left[\frac{1-\cos(\phi_0-\phi)}{\phi_0} + \left(1+\frac{t^2}{3\,r^2}\right)\frac{\phi_0-\phi}{2\sin\phi_0}\sin\phi - \frac{1}{2}\left(1-\frac{t^2}{6\,r^2}\right)\sin(\phi_0-\phi)\right] p\,(\phi)\,d\phi = \int_0^{\phi_0} p\,(\phi)\,S\,(\phi)\,d\phi \ldots (196)$$

in which, $R\,(\phi)$ and $S\,(\phi)$ are abbreviations that represent well-defined and given functions of ϕ. The bending moments that occur along the arch segment at any section created by the reactions, M_0 and H_0, as well as the assumed symmetrical load, $p\,(\phi)$, may be expressed mathematically, as follows:

$$M = \int_0^{\phi_0} p\,(\phi)\,R\,(\phi)\,d\phi + r\left(\cos\phi - \frac{\sin\phi_0}{\phi_0}\right)\int_0^{\phi_0} p\,(\phi)\,S\,(\phi)\,d\phi + r^2\int_{\alpha=0}^{\alpha=\phi} p\,(\alpha)\sin(\phi-\alpha)\,d\alpha \ldots\ldots\ldots\ldots\ldots (197)$$

Equation (197) must be equal to a prescribed moment, say, $r^2 F(\phi)$. The equation may be simplified by introducing new signs for the purpose of condensation. Thus, Equation (197) may be written in the following form:

$$\int_0^{\phi_0} p\ (\phi)\ U\ (\phi)\ d\phi + \cos\ \phi \int_0^{\phi_0} p\ (\phi)\ V\ (\phi)\ d\phi + \int_{\alpha=0}^{\alpha=\phi} p\ (\alpha)\ \sin\ (\phi - \alpha)\ d\alpha = F(\phi) \ldots\ldots\ldots\ldots (198)$$

This formula differs greatly from that which connects load and deflection functions, and it may be solved quite rapidly because of the indefinite form of the integral in the third term.

In order to solve Equation (198) the derivatives of that term must be determined. It is a composite function of ϕ inasmuch as the integral is an explicit function of ϕ as well as α. By use of the calculus the derivative of such a function may be obtained by means of the general formula,

$$\frac{d}{dx} \int_0^x g\ (x, \alpha)\ d\alpha = g\ (x, x) + \int_0^x \frac{\delta}{\delta\ x}\ g\ (x, d)\ d\alpha \ldots\ldots\ldots (199)$$

Hence, the first derivative of Equation (198) will be,

$$-\sin\ \phi \int_0^{\phi_0} p\ (\phi)\ V\ (\phi)\ d\phi + p\ (\phi) \sin\ (\phi - \phi) + \int_{\alpha=0}^{\alpha=\phi} p\ (\alpha) \cos\ (\phi - \alpha)\ d\alpha = \frac{d}{d\phi} F(\phi) \ldots\ldots\ldots\ldots (200)$$

since the integrals in the first two terms of Equation (198) are definite integrals and are thus constant values. The second term of Equation (200) is, of course, equal to zero. It has been included in the equation only to clarify the operations involved in the analysis.

A second differentiation is needed, thus,

$$-\cos\ \phi \int_0^{\phi_0} p\ (\phi)\ V\ (\phi)\ d\phi + p\ (\phi) - \int_{\alpha=0}^{\alpha=\phi} p\ (\alpha)\ \sin\ (\phi - \alpha)\ d\alpha = \frac{d^2}{d\phi^2} F(\phi) \ldots\ldots\ldots (201)$$

By adding Equations (198) and (201), the two functional terms will drop out with the result that,

$$\int_0^{\phi_0} p\ (\phi)\ U\ (\phi)\ d\phi + p\ (\phi) = F(\phi) + \frac{d^2}{d\phi^2} F(\phi) = g\ (\phi) \ldots. (202)$$

Since the first term of Equation (202) is constant, the unknown function, $p\ (\phi)$, must be practically equal to $G\ (\phi)$. By substituting C instead of the first term, the following relationship is determined:

$$p\ (\phi) = G\ (\phi) - C \ldots\ldots\ldots\ldots\ldots\ldots (203)$$

in which, $G\ (\phi)$ is a well-known function. Substituting this value of $p\ (\phi)$ into the first term of Equation (202), the following relation is obtained:

$$C = \int_0^{\phi_0} [G\ (\phi) - C]\ U\ (\phi)\ d\phi = \int_0^{\phi_0} G\ (\phi)\ U\ (\phi)\ d\phi - C \int_0^{\phi_0} U\ (\phi)\ d\phi \ldots\ldots\ldots\ldots\ldots (204)$$

Whence,

$$C = \frac{\int_0^{\phi_0} G(\phi)\, U(\phi)\, d\phi}{1 + \int_0^{\phi_0} U(\phi)\, d\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (205)$$

Thus, the final expression for $p(\phi)$ will be,

$$p(\phi) = F(\phi) + \frac{d^2}{d\phi^2} F(\phi) - \frac{\int_0^{\phi_1} \left[F(\phi) + \frac{d^2}{d\phi^2} F(\phi) \right] U(\phi)\, d\phi}{1 + \int_0^{\phi_0} U(\phi)\, d\phi} \quad \ldots\ldots (206)$$

This value of $p(\phi)$ is to be substituted in Equation (198). To return to the original problem, it is possible, by making use of Equation (204), to find a satisfactory expression for $f(y, \phi)$, so that the arch slice at the bottom, acted upon directly by the load, $w - f$, and the transverse bending moments, M, resulting from the cantilevers subjected to the load, f, and indirectly by the hypothetical horizontal bending moments, will not undergo any elastic deformation.

In this manner the stresses produced in the arches and cantilevers may be determined and the integral of work may be evaluated. Likewise, also, the most probable value of the parameters may be found. An alternate procedure may be outlined as follows: It is possible to determine the most satisfactory values of the parameters by applying formulas and neglecting the influence of the moments, $\frac{\mu M}{E}$, on the arches. The parameters thus obtained will define with a fairly good approximation the true conditions as to the behavior of M in the lower parts of the dam.

The stresses used in the preceding equations, as expressed by means of the general parameters, may be completed by assuming that stresses arise in the lower parts in consequence of the moments, $\frac{\mu M}{E}$. The entire procedure of determining the parameters may be repeated. The results thus obtained will make possible a greatly improved solution. The influence of changes in temperature or shrinkage, or the yielding of the foundation, may be treated in a similar manner.

The writer does not think it necessary to present any numerical computations to illustrate this solution. A method has been pointed out which permits the analyses of the dam when considered as a shell. In the writer's opinion it is grasped comparatively easily by engineers because it appeals to them from a practical standpoint.

Some comments have been made as to the probable distribution of the different kinds of stresses in the shell. These may be more or less difficult to follow at first, but a little thought will clarify them.

After becoming acquainted with the fundamental concepts of the correcting or completing stresses or load systems that act on the arch slices in addition to the water pressure, there can be no further difficulty in grasping the method

described herein. The problem becomes reduced to a simple computation of stresses in beams and arches acted upon by a given load. Such problems are familiar enough to engineers whose primary interest is in the design of dams.

When stresses are computed and expressed as functions of the parameters, the final computation of the integral of work and of the resulting system of linear equations for the parameters is very simple and easy in operation. There may be considerable material work, but no appreciable difficulty.

To conclude, the proposed method consists of four steps, which are as follows:

(1) Choose a convenient correcting load containing a few parameters so that it acts in addition to water pressure on the isolated arch and cantilever slices.

(2) Compute the stresses caused in the arches and cantilevers by the load thus chosen.

(3) Evaluate the integral of work expressed in the stresses acting in the arch and cantilever slices, as computed in Step (2). Derive the equation for the parameters from the condition that they should minimize the work of deformation.

(4) Solve the resulting linear equations for the parameters.

The last three steps require no mental effort. The accuracy and the reliability of the estimation required for a satisfactory solution of the problems involved in the first step would be greatly improved by experimentation on a few typical shapes of the shell-canyon profiles on small models.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EFFECT OF TURBULENCE ON THE REGISTRATION OF CURRENT METERS

Discussion

BY FORREST NAGLER, M. AM. SOC. C. E

FORREST NAGLER,[1] M. AM. SOC. C. E. (by letter).[1a]—Exhaustive investigation on the effect of turbulence on current meters has long been lacking, and the paper under discussion fills a very definite gap in water-measurement practice. The work is exceptionally valuable, not only because of the methods used, but also because of the wide variety of current-meter forms tested. The meter performances investigated have to deal with solid angles, although they must be analyzed in horizontal and vertical planes for the purpose of showing their characteristics in diagrammatic form. It seems, however, that a polar diagram is slightly more appropriate than the rectangular diagram used by the authors for showing the detail meter characteristics.

Figs. 29 to 34 are submitted to show a line of investigation carried out by the writer, that parallels with surprising closeness a small portion of the work covered in the paper. Figs. 29 to 32, inclusive, show the detail work done on some of the meters available.

The initial experiments were made on a large sized Price meter in excellent condition. The curves in Fig. 29 show how it is affected by flows at angles with the horizontal. On account of the obvious inaccuracy of the meter in question, the work was not carried out farther than indicated in the diagram. Later, in order to obtain an approximate comparison of the large Price meter with other meters, an arithmetical average was made between the vectors of Curve *D*, Fig. 29, and Curve *B*, Fig. 30, the latter being made on a small-sized, cup type of meter. The varying effect of different forms of frame will undoubtedly influence the form of Curve *B*, which is made in a horizontal direction, but for approximate purposes it was considered to be sufficiently accurate. The large Price meter was the least accurate of all those examined, and was particularly inaccurate if there is a preponderance of flow coming at an angle

NOTE.—The paper by David L. Yarnell and Floyd A. Nagler, Members, Am. Soc. C. E., was published in December, 1929, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: February, 1930, by Charles S. Bennett, M. Am. Soc. C. E.; May, 1930, by Messrs. Brehon B. Somervell, N. C. Grover, Burke L. Bigwood, G. H. Nettleton, and C. Linden; August, 1930, by H. R. Leach, M. Am. Soc. C. E.; September, 1930, by Dr.-Ing. Ludwig A. Ott; and November, 1930, by B. F. Groat, M. Am. Soc. C. E.

[1] With A. O. Smith Corporation, Milwaukee, Wis.

[1a] Received by the Secretary, January 24, 1931.

with the horizontal. It registers a velocity in excess of the desired resolved components under all conditions.

The small Price meter (Fig. 30) shows up unusually well up to angles of 25°, but it will be noted that this applies only for conditions of angular flow evenly distributed, horizontally and vertically. For such a condition this meter is probably very accurate, but it ceases to be so the moment there is a preponderance of angularity either in the horizontal or the vertical direction. For example, if the preponderance is in a horizontal direction, the meter registers an excess velocity; and, if it is vertical, it registers less than the true amount up to about 40 degrees.

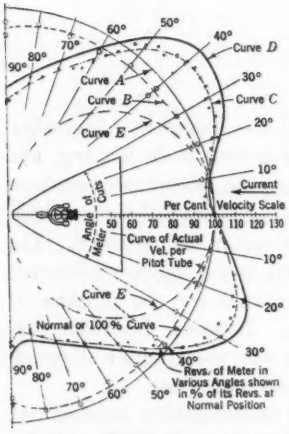
Curve D
Curve A
Curve B
Curve C
Curve E
Angle of Cups
Per Cent Velocity Scale
Curve of Actual Vel. per Pitot Tube
Curve E
Normal or 100 % Curve
Revs. of Meter in Various Angles shown in % of its Revs. at Normal Position

FIG. 29.—TESTS WITH A LARGE PRICE METER.

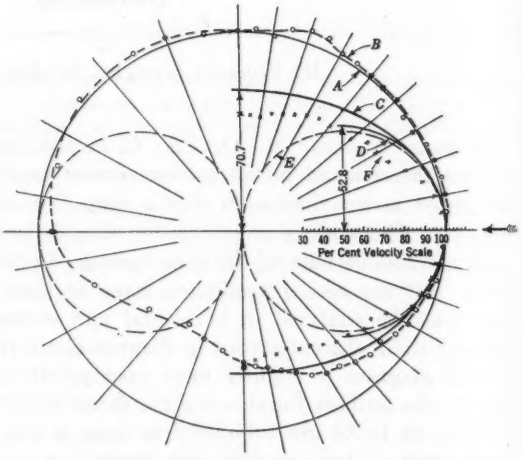

FIG. 30.—TESTS WITH A SMALL PRICE METER.

Examination of the results obtained with the screw type of meter shows that it is the ideal form for use in conditions of disturbed flow, because it makes the same effort to give resolved components regardless of the direction of the angularity. Of the two meters examined (see Figs. 31 and 32), the Ott meter is considerably the better. Incidentally, it may be mentioned that this applies not only to performance, but to the mechanical structure of meter bearings and housing. On the basis of the correction to be applied to Curve 4 in Figs. 33 and 34, making its semi-minor axis 35 instead of 32, it becomes evident that these two meters show results not differing widely, but in the same direction from a true value.

The various meters were mounted on a vertical or horizontal axis on a frame installed in a 42-in. pipe in which water at a temperature of between 90 and 100° Fahr. flowed to a depth of about 14 in., at a velocity of approximately 4 ft. per sec. This velocity was very constant, the discharge being from several large condenser pumps. The pipe was of concrete, uniform in section, fairly smooth inside, and several hundred feet long, with constant slope. The flow was even and practically parallel by reason of the regular section, high velocity, and long smooth channel. Pitot tube readings were

taken simultaneously with meter reading both at the zero position and at the angular positions so as to show whether the movement of the meter affected the flow. The tube was used for this purpose rather than to check the actual velocity reading of the meter, although this latter comparison is available.

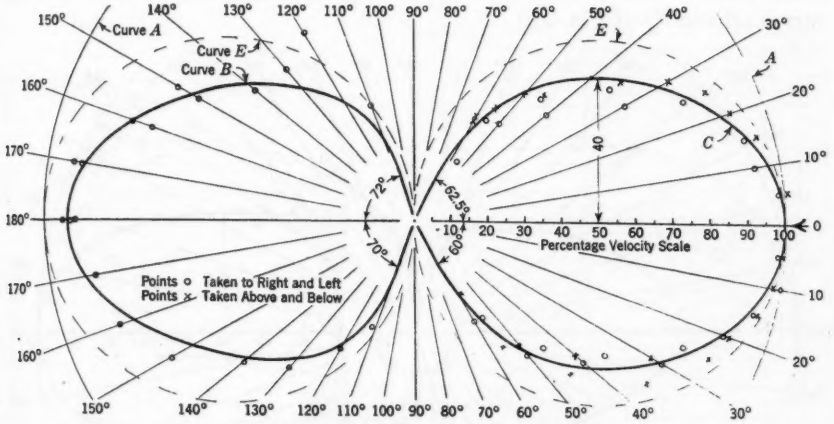

FIG. 31.—TESTS WITH AN OTT METER.

In plotting results of the tests the curves could not be fixed with sufficient accuracy by inspection, so the method of least squares was adopted for determining the probable location and form of characteristic curve. This was used for all meters, except the large Price, which was handled entirely by inspection and by an arithmetical average of the vectors of the two curves.

On the basis of the foregoing determination, the following semi-minor axes were determined:

Small Price,	$B = 70.7$
Ott,	$B = 40$
Haskell,	$B = 32$

The determination for the small Price meter was also made on the basis of Curves D and $2D$, shown in Figs. 30 and 33, in order to make possible a direct comparison of the probable characteristics of the three meters. The various ellipses determined on the foregoing basis are shown in Figs. 29 to 32, in which the circles, A and E, are the two basic curves. To afford better comparison these various curves are replotted together in Fig. 33. Bringing together all curves in this form suggests a method of correcting differences between any two meters. In order to facilitate this comparison, the separate curves may be defined as follows:

Curve 1 represents the performance of a large Price meter rigidly supported and subjected to angular flow uniformly distributed. (If the preponderance is known or suspected to be in either the vertical or horizontal direction, reference, respectively, to Curve D (Fig. 29), and Curve E (Fig. 30), will indicate the direction and possible magnitude of error.)

Curve 5 represents a 100% curve, which would be expected from any good make of meter so supported as to be free to turn and measure the thread of the

current, provided the support be such that the revolving element is given no sidewise motion.

Curve 2*C* represents the performance of the small Price meter rigidly supported and subjected to angular flow uniformly distributed. (The note given under Curve 1 applies here, referring to the horizontal and vertical curves (*B* and *F*) of Fig. 30.)

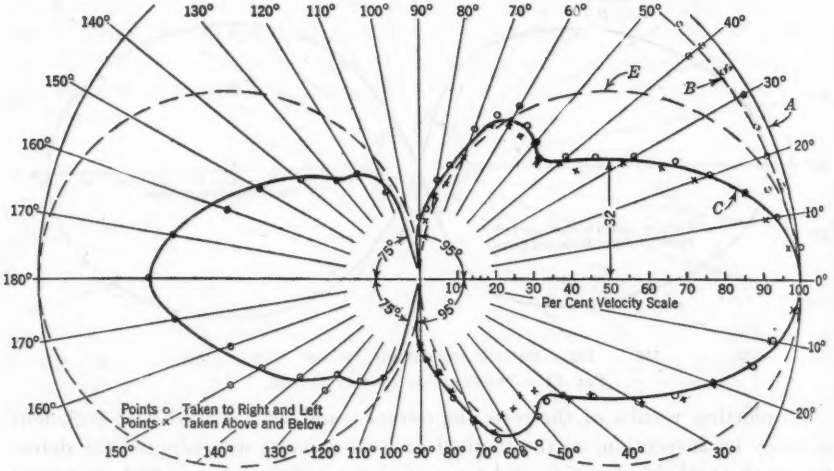

FIG. 32.—TESTS WITH A HASKELL METER.

Curve 2*D* represents a forced mathematical interpretation of the performance of a small Price meter made for the purpose of comparison only, and should not be used hereafter.

Curve 0 shows the ideal or cosine curve. If the meter had this characteristic it would always register only the desired component; hence it would be ideal (inertia effects neglected).

Curve 3 represents the performance of the Ott, screw type of meter.

Curve 4 represents the performance of the Haskell, screw type of meter.

Referring to Figs. 33 and 34, the average percentage excess registered by the Price meter is equivalent to the volume of the solid of revolution (*A*), divided by the area of that portion of the surface of the sphere the intercept of which is the Curve 0, enclosed by a cone with an apex angle of 40 degrees. Similarly, the average percentage deficiency registered by the Ott is obtained by dividing the volume of the solid of revolution (*B*) by the same area. The determination of these averages would serve as a basis for reconciling the discrepant results obtained by the two meters in actual work. Similar determinations for the various other meters can readily be made.

In forming the computations, integration is difficult by reason of the complicated forms of some of the surfaces, but a simpler method is available. Fig. 34 is plotted directly from Fig. 33, the radii vectors of the various curves being plotted as percentages of the corresponding radii vectors of the zero circle for angles up to 45 degrees. These curves are plotted according to the

small percentage scale at the left of Fig. 34. On this diagram the areas, A and B, and, consequently, their respective volumes of revolution about the vertical axis, correspond to the similar areas and volumes for Fig. 33. Furthermore, the area of the surface described previously is now determined as the area of a circle of radius (20) in Fig. 34.

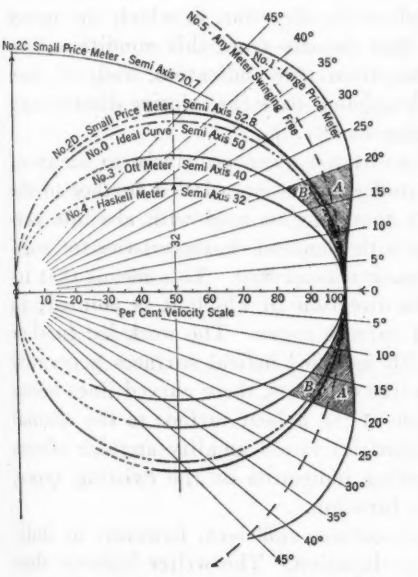

FIG. 33.—SUMMARY OF CONTRASTING METER CHARACTERISTICS.

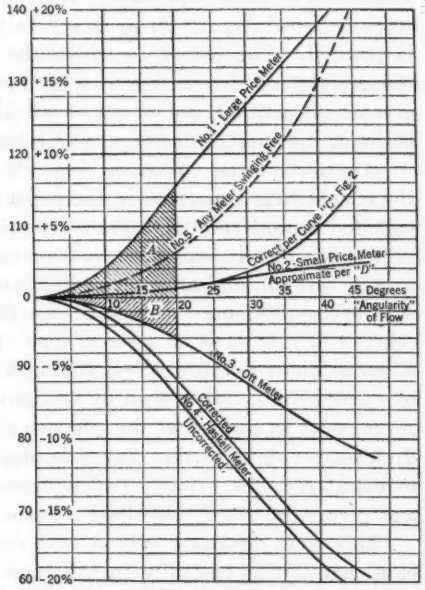

FIG. 34.—ALTERNATE RECOMMENDATION FOR PLOTTING FIG. 33.

By inspection of Fig. 34 it is seen that the various curves are approximately parabolic in form, particularly near the origin. Therefore, the various volumes may be determined by multiplying the areas of the base of the solid of revolutions by one-half its height. Dividing this volume by the area of base, in turn, leaves a mean ordinate of one-half the height of the actual ordinate of the curve. This permits of a simple solution by dividing the vertical percentage scale by 2, resulting in the large percentage scale indicated at the left as plus and minus.

The writer believes that the plotting shown in Fig. 34, while being much less complete than that shown by Fig. 17, possesses a feature which could desirably be introduced into the latter. Fig. 17 apparently averages for any particular stated angle, all the variations of angular flow both horizontal and vertical, but at that angle only. Fig. 34 is basically an integrated diagram and the plotted points of the curves at any angles (say, 20°) are not an average of angular effects at 20°; they are a summation or average of all the effects from zero to 20°, assuming that these effects exist a uniform length of time for all degrees up to that maximum. It is an attempt to define turbulence as being represented by constantly varying impact angles within and up to a

certain maximum. This appeals to the writer as being a truer picture of turbulent flow and its angularity effect on a current-meter head.

The writer's work, done in 1913 and 1914, was premised on the assumption that all discrepancies in current-meter observations could be traced to disturbed conditions in the flow, and that all types of meters would register alike according to their still-water ratings if the flow were of stream-line characteristics, straight, uniform, and parallel to the direction in which the meter is pointed. The extent to which the flow departs from this condition fixes the amount of divergence of any meter from true indication, and, of even greater importance, the extent of the disturbance fixes the relative discrepancy between the indications of any particular meters.

Analysis of the curves in Figs. 29 to 34 led quite positively to many of the conclusions arrived at in the paper under discussion. It led further to the conclusion that the only style of meter that may be used with absolute certainty, is a rigidly supported instrument with identical characteristics for horizontal, vertical, and all intermediate angularities of flow. This means that its axis of rotation must coincide with the direction in which it is pointed; in other words, it must be a screw type of current meter. The work led further to an attempt to build screw meters with modified helical surfaces generated by curved lines, rather than by straight-line elements, these curved lines being so disposed as to present the concave side of the helical surface to the normal direction of flow, so that any side angularities would amplify angular effects to counteract the evident under-registering indication of the existing types, and give more nearly the desired cosine function.

This work attained only a moderate success, sufficient, however, to indicate that it was basically in the correct direction. The writer believes that such a meter can be built very much along the lines of a small Ott meter, but with a more nearly correct cosine response to angular flow. It has been demonstrated that it is simple to increase the semi-axis of the small Ott diagram shown in Fig. 31, to such an extent that positive readings are obtained at angularities of 70 and 80 degrees. It was further demonstrated that the adherence of all points to the ideal cosine curve, E, is a question not only of curved generating elements, but also of the shape of the head itself, Fig. 32, illustrating a peculiarity evidently traceable to shape of head. The complete development of this combination presents an excellent field for improved present current-meter types.

Fig. 34 is presented as a means of reconciling discrepancies found in using meters of all different types. The discrepancy between meters of known characteristics is an indication of the degree of turbulence or angularity encountered. What is of greater importance, it might also offer a clue to the correct flow even if that flow is different from that indicated by either of the meters. In such application it is desirable to have the two meters under comparison, of diverse characteristics, so that correct flow will lie between them. This is merely because it takes quite a stretch of the imagination to use two such meters as the Ott and the Haskell (which, at an indicated angularity of 30 in Fig. 34, may under-register, respectively, 6% and 11½%), and increase the discharges of both of them by the amount in question merely

because they happen to differ by about 5½ per cent. Such a procedure, however, is in line with a logical analysis of the data provided in the paper and in this discussion.

An investigation of completely shrouded meters would be very interesting. The shrouded Fteley meter analyzed[2] by Charles P. Rumpf, M. Am. Soc. C. E., shows an approach to the true cosine circle (Curve *A*), indicated by a semi-minor axis of about 45 units. Similar analysis of the large shrouded screw meter made by Gurley would be of interest.

To the writer the significant showing of the paper under discussion is in the extreme over-registering possible with both screw and cup type of meters under turbulent conditions. The under-registering of the Haskell type under good, but angular, flow conditions is the sole contrast, but it should be pointed out that under-registering resulting in a turbine efficiency exceeding 100% (as has actually happened) is instantly discarded; whereas, a result of 80% caused by over-registering carries no red warning flag labeled "impossible".

The authors are contributing a most excellent means of judging relative meter performance, to the end that wide inaccuracies in either direction may be prevented.

[2] *Engineering News*, Vol. 71, No. 20, May 14, 1914, p. 1083.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion

BY FRANCIS P. WITMER, M. AM. SOC. C. E.

FRANCIS P. WITMER,[10] M. AM. SOC. C. E. (by letter).[10a]—The author has described his method of analysis so clearly that its virtues are obvious. The writer believes that its simple underlying principles and surprisingly easy application make it the most convenient and expeditious method yet brought forward for the practical solution of numerical problems in internal bending stresses. For algebraic solutions it is, of course, not practically applicable as, without numerical values, one could not know when the process had been carried far enough for proper approximation.

The writer would like to comment on the subject of algebraic signs for moments. In 1917 he had occasion to make an investigation of indeterminate bending stresses resulting from eccentric connections in the trusses of an old elevated railroad structure. This led him to develop a method of analysis which has since become familiar as the "slope-deflection method". The necessity was quickly apparent for assuming that all moments in the same rotational direction were positive, regardless of where they were applied or whether they were external moments or internal resisting moments. Any other assumption was practically certain to result in errors and inconsistencies.

For illustration, consider the frame in Fig. 33. Assume that the various members have center bending from force action, as shown by the curved, dotted lines. The usual convention, as adopted by Professor Cross, would naturally lead to the algebraic designations shown for moments in all members except BC; but how shall moments in these members be designated? As they are nearly vertical, it may be logical to consider them similarly to the columns, CD; but this would be inconsistent with the adjacent members, AB, which are certainly analogous, structurally, to the members, BC. The adopted convention must not have limitations depending upon the angle of slope of

NOTE.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in May, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by Messrs. C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin and E. F. Bruhn; October, 1930, by Messrs. A. H. Finlay, R. F. Lyman, Jr., R. A. Caughey, Orrin H. Pilkey, and I. Oesterblom; November, 1930, by Messrs. Edward J. Bednarski, S. N. Mitra, Robert A. Black, and H. E. Wessman; January, 1931, by Messrs. Jens Egede Nielsen, F. E. Richart, and William A. Oliver; and February, 1931, by Messrs. R. R. Martel and Clyde T. Morris.

[10] Director of Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

[10a] Received by the Secretary, January 14, 1931.

members. Dilemmas such as these will continually occur in computations of this character, unless the convention adopted permits the assumption that all moments in one rotational direction are positive.

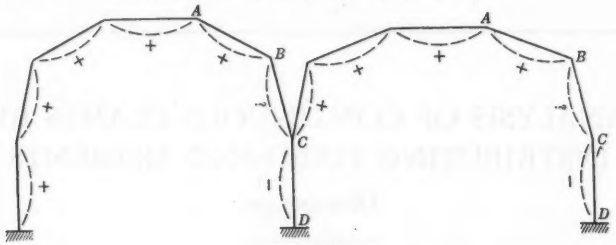

FIG. 33.

Professor Cross has very properly avoided giving proofs for each of the three operations which constitute his method. The writer cannot refrain, however, from a brief statement which he believes will place these relations in a very clear light, for cases in which the moment of inertia is constant throughout the length of each member.

The fundamental expression for the slope-deflection method of analysis, in cases where the straight-line axis joining the ends of a member does not change position, is

$$M_1 = \frac{2\ E\ I}{l}(2\ \theta_1 + \theta_2)\ +\ C \qquad (41)$$

in which, C is a constant depending upon the particular loading. All the relations required in Professor Cross' method may be readily derived from Equation (41), as follows:

First.—For a beam fixed at both ends, the end, bending-deflection angles, θ_1 and θ_2, will both be zero, and $M_1 = C$. In other words, the fixed-end moment at either end is simply equal to the constant in the slope-deflection equation which corresponds to the existing loading. Tables of these constants are available in recent books describing this method of analysis.

Second.—Assume a moment applied at one end of a member, the other end being fixed. From the action of this moment alone (remembering that θ_2 at the fixed end is zero), Equation (41) becomes $M_1 = \frac{4\ E\ I\ \theta_1}{l}$. If two or more members meet and are rigidly connected at a point, and a total moment, M, is applied at this point, M will be so divided among the members that the ends of all will deflect through the same angle, θ_1, and these members, therefore, will carry moments proportioned thus:

$$M_1 : M_2 = 4\ E\ \theta_1\ \frac{I_1}{l_1} : 4\ E\ \theta_1\ \frac{I_2}{l_2}$$

or,

$$M_1 : M_2 = \frac{I_1}{l_1} : \frac{I_2}{l_2};\ M_1 : M_3 = \frac{I_1}{l_1} : \frac{I_3}{l_3};\ \text{etc.}$$

This proves the relation used in the second operation.

Third.—Assume, again, a member with a moment, M_1, at one end and a fixed moment, M_2, at the other end. From Equation (41),

$$M_1 = \frac{2\,E\,I}{l}\,(2\,\theta_1 + \theta_2)$$

and,

$$M_2 = \frac{2\,E\,I}{l}\,(\theta_1 + 2\,\theta_2)$$

Since θ_2 at the fixed end is zero, $M_1 = \frac{4\,E\,I\,\theta_1}{l}$, and $M_2 = \frac{2\,E\,I\,\theta_1}{l}$. Hence, $M_2 = \frac{1}{2}M_1$, and the "carry-over" factor is $+\frac{1}{2}$, as assumed in the third operation (with the proposed modification in the convention as to signs).

These relations are well known, but it seemed to the writer worth while to direct attention to the ease with which all the necessary relations may be established from the single expression, Equation (41).

The author mentions the possibility that an unsymmetrical condition of form or loading may cause the structure to sway, and indicates a process by which correction may be made to allow for this. It appears to the writer that such a condition is really the normal one, and that the case without sway is exceptional. It seems to him important that the process of correction be illustrated by simple numerical examples, in order to increase the availability of Professor Cross' method of analysis.

Consider, first, a frame without any applied horizontal force, but one which is unsymmetrical both in form and loading, as in Fig. 34(*a*). The correct moments and reactions, as computed by the slope deflection method, are shown. Fig. 34(*b*) shows the same frame with moments computed by Professor Cross' method, assuming that no sway occurs. From the moments in the posts, the shear reactions, H_1 and H_2, are computed to be of the values shown. Since these are unequal numerically, equilibrium does not obtain, showing that the structure has sway. The force, H_1, must be increased and the force, H_2, decreased so that they may equal each other, for equilibrium. Furthermore, the numerical sum of these two correcting quantities must be equal to the numerical difference between H_1 and H_2 as first obtained and the two corrections must be proportioned to the values of $\frac{I}{l^3}$ for the respective columns, in order to equalize their sway. By observing these conditions it is found that H_1 must be increased by 17.1 lb. and H_2 must be decreased by 63.3 lb., thus bringing the resulting reactions to agreement at the value, 103.3 lb. This is practically the same as in Fig. 34(*a*).

The moments in the posts must now be corrected to correspond with these corrections in shear. The total correction for the left-hand post will be, $17.1 \times 1\,000 = 17\,100$ in-lb. and that for the right hand post, $63.3 \times 600 = 37\,980$ in-lb. These total correcting moments must be properly divided between the top and bottom of the posts. If the horizontal girder is infinitely rigid the tops will be perfectly fixed by these correcting moments, and the points of contraflexure produced thereby will be at mid-height, thus dividing

the correcting moments equally between top and bottom. With this assumption for the present case, the resulting total moments will be:

Left post:

At the top, 66 100 in-lb.; at the bottom, 37 300 in-lb.

Right post:

At the top, 47 700 in-lb.; at the bottom, 14 300 in-lb.

These are very close to the values in Fig. 34(*a*), as they should be, since the girder is very rigid, as compared with the columns.

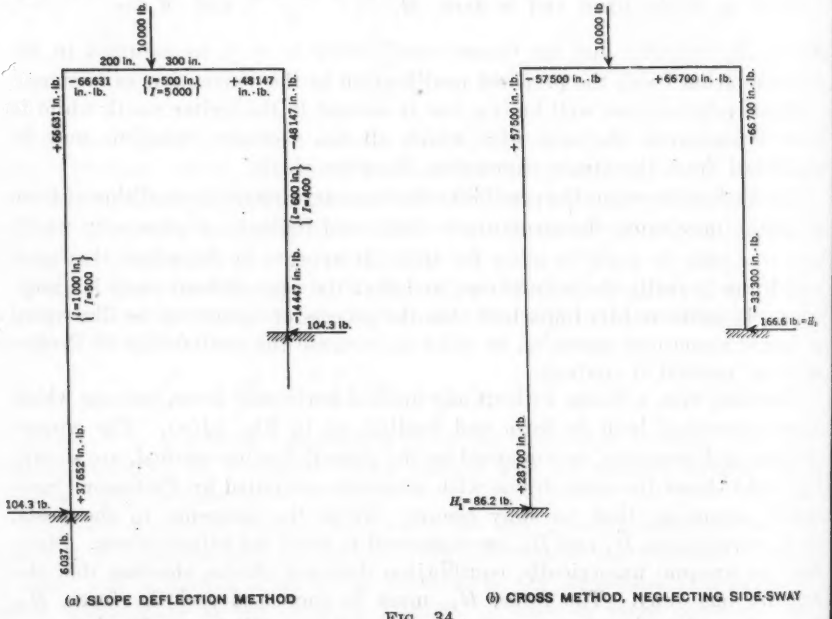

FIG. 34.

If the girder had no rigidity, the points of contraflexure, due to the correcting shears, would be at the top and the moment corrections would be wholly at the bottom. All cases will fall between these two extremes, and the correct proportion must be assumed by consideration of the relative rigidities of the girder and column. Probably, in most cases, the true corrections will not be far from equal at top and bottom, and, undoubtedly, the corrected moments may always be found in this manner, with a reasonably close approximation to the true values.

Consider, next, a frame with an applied horizontal force which will necessitate side-sway, whether or not the frame is symmetrical. Let Fig. 35(*a*) represent such a case, in which there also exists a lack of symmetry in size of columns. The moments and reactions shown on Fig. 35(*a*) are correct as computed by the slope-deflection method. First, divide the horizontal force between the two columns in proportion to their values of $\frac{I}{l^3}$, in order to equalize their sway, obtaining 3 333 lb. at the left and 6 667 lb. at the

right column. Next, compute end moments for the columns, assuming them to be fixed top and bottom, following this by distributing unbalanced moments, according to the Cross method. Fig. 35(*b*) shows the resulting moments and forces. It is seen that the horizontal forces are not in equilibrium, showing that there has been sway in addition to that naturally resulting from the applied horizontal force; that is, due to lack of symmetry in the frame. If vertical loads had been applied in an unsymmetrical manner, these also would have contributed to this sway.

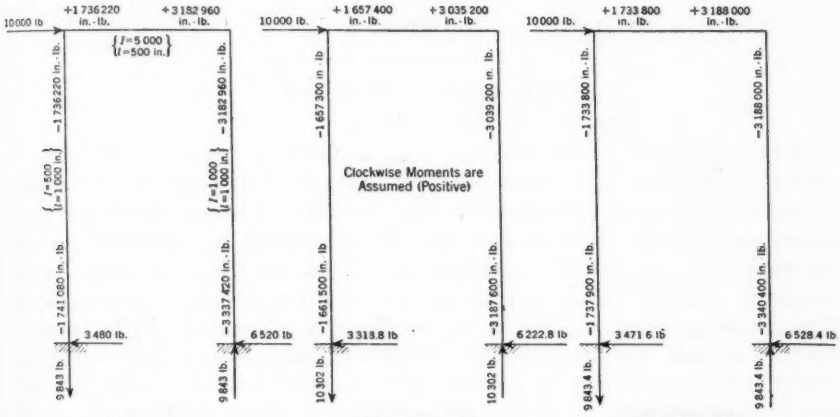

FIG. 35.

The amount of unbalanced horizontal force is found to be 458.4 lb. acting toward the left. Each horizontal reaction, therefore, must be increased in proportion to the value of $\frac{I}{l^3}$ of its column, resulting in corrections as follows:

152.8 lb. at the left column; and 305.6 lb. at the right column. Adding these to the horizontal reactions formerly obtained (see Fig. 35(*b*)), will balance the horizontal forces. The total moments for the columns must now be corrected by the following:

Left column ..... 152.8 × 1 000 = 152 800 in-lb.
Right column ..... 305.6 × 1 000 = 305 600 in-lb.

If the tops are assumed to be fixed by the girder, these moments will be equally divided between top and bottom, and the resulting moments and forces will be as shown in Fig. 35(*c*). These are seen to check very closely with the correct values in Fig. 35(*a*). If the girder is relatively less rigid, the point of contraflexure will rise and the bottom correcting moment will be greater than that at the top, but generally the assumption of equal corrections at top and bottom will probably be sufficiently accurate.

The methods of correction just described may readily be applied to a building of several stories, by the exercise of a little ingenuity, and thus it is probable that the wind moments in a tall building may be computed with sufficient accuracy, and with far less expenditure of time and labor than by any of the methods heretofore available.

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DISCUSSIONS

EVAPORATION AS A FUNCTION OF INSOLATION

Discussion

BY BURT RICHARDSON, ESQ.

BURT RICHARDSON,[1] ESQ. (by letter).[1a]—Much interest has been shown in this study of evaporation of lakes, and the writer was gratified to receive inquiries regarding the possibility of computing the evaporation quantitatively from a knowledge of insolation. As might be expected the Western States have shown the most interest in the subject. For instance, the Forestry Department of the University of California has made inquiries; graduate students of the University of Southern California are repeating the experiment described in the paper; the United States Naval Air Station, at San Diego, Calif., is interested in a fog-prediction method as well as evaporation and insolation data; the Scripps Institution of Oceanography of the University of California, at La Jolla, is developing a plan for rainfall prediction for Southern California, in which evaporation and insolation are two important quantities; while the motion picture industry and the Stanford University Marine Biological Station welcome the insolation and atmospheric absorption data.

The evaporation from a heat-insulated tank can always be computed with excellent results when the experiment is made with a pyrheliometer, good instruments, and good atmospheric data such as those from the Mount Wilson Observatory; but when the quantity of evaporation is from a lake, say, in Beesville, Tex., where no pyrheliometer station is located, where the absorption coefficient has never been computed, and where the observed value of evaporation has been measured by some caretaker of the lake—then the results must of necessity be rough average approximations, as suggested by Dr. Abbot. However, limiting values of each term used in the evaporation formula can be selected, and their use will set definite limits to the quantity of evaporation. In any event, these evaporation estimates should be more accurate than applying to a lake a method which is based on a knowledge of the rainfall and an estimate of the quantity of inflowing and outflowing water with a correction for seepage.

NOTE.—The paper by Burt Richardson, Esq., was published in May, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by Messrs. C. G. Abbot, and H. H. Kimball; and October, 1930, by Messrs. D. P. Krynine, R. C. Wells, and N. W. Cummings.

[1] Prof., Dept. of Physics, Occidental Coll., Los Angeles, Calif.

[1a] Received by the Secretary, February 16, 1931.

The mathematical "short-cuts" suggested by Professor Krynine are quite welcome. According to information received from the Mount Wilson Observatory, the values of the two terms, A and ρ, used in this paper are not known for places in the United States other than those mentioned. The method of making observations of temperatures of water surface, wet-bulb, and dry-bulb, which are required in computing the terms, H, P_w, T_a, and T_w, is rather fully described in the two references to the *Physical Review*.

No difficulty presents itself in applying the evaporation formula to a lake during the night when, of course, the insolation is equal to zero. At that time, if the water is warmer than the air, the heat that goes into evaporation, as well as the radiation, conduction, and convection, is taken, calorie for calorie, from the quantity of sensible heat stored in the body of the water.

Professor Cummings[2] has recently done some experimental work which shows clearly the rate of evaporation from tanks of various sizes. He has also shown the relation of evaporation to humidity and convection.

Since presenting this paper the writer has computed the evaporation from most of the lakes of the United States where such evaporation had been observed, and the results have been satisfactory. The best results obtained were in cases where the lakes were near pyrheliometer stations.

The writer is convinced of the importance of evaporation studies and will continue to urge those who are interested in the storage of water to gather and publish data regarding observed evaporation and temperatures, which have been made in a uniform manner. He also will continue to urge those institutions that are financially able to do so to install Weather Bureau pyrheliometers; and he will attempt to interest astronomers and meteorologists in studying the absorption coefficient of the earth's atmosphere.

[2] "Relation between Evaporation and Humidity", by N. W. Cummings, *Bulletin*, National Research Council, No. 68 (1929), pp. 47-56; also, "Alignment Diagram for Ratio of Convection to Evaporation and Limits of Ratio of Convection to Evaporation," by N. W. Cummings, *Monthly Weather Review*, Vol. 58 (1930), pp. 142–146.

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DISCUSSIONS

SOME ASPECTS OF WATER CONSERVATION

Discussion

BY JOHN W. PRITCHETT, ASSOC. M. AM. SOC. C. E.

JOHN W. PRITCHETT,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—The author's estimate of total storage for irrigation already developed or proposed by the U. S. Bureau of Reclamation does not include proposed reservoirs in Texas which have not been investigated by the Federal Reclamation Bureau. The developed storage for irrigation in Texas is more than 1 000 000 acre-ft., not including the Lower Rio Grande Valley, where permits have been granted for the storage of about 275 000 acre-ft. Surveys and estimates have been made of sites that show favorable opportunities for the development of more than 3 500 000 acre-ft. of storage. The storage developed for water power in Texas amounts to about 25 000 acre-ft. Storage being developed for water power, or for which State permits have been granted for water power, are equal to more than 2 500 000 acre-ft. The storage developed or being developed for municipal water supply equals 900 000 acre-ft. That being developed for flood control in connection with other uses amounts to 500 000 acre-ft. In addition to the foregoing, a preliminary district has been created by the Legislature, looking toward the development of the entire Brazos River water-shed for irrigation, municipal supply, power, and flood control. The area of this water-shed is about 40 000 sq. miles, and the mean annual run-off is 5 500 000 acre-ft. The reservoir sites that have been surveyed in this water-shed give a total storage capacity of more than 4 500 000 acre-ft.

Some of the common errors noted by the author, in the investigation and development of water conservation, are among those generally encountered in the investigations in this section of the United States, namely:

1.—The use of insufficient data on stream flow.
2.—The use of rainfall records and assumptions as to run-off without adequate data showing the relationship between them.
3.—An under-estimation of flood flows, and insufficient spillway provision.

NOTE.—The paper by R. A. Sutherland, Assoc. M. Am. Soc. C. E., was published in September, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: January, 1931, by Messrs. E. C. Eaton and A. W. Simonds; and February, 1931, by Messrs. W. H. R. Nimmo and C. R. Olberg.

[9] Engr., State Board of Water Engrs., Austin, Tex.

[9a] Received by the Secretary, December 19, 1930.

TABLE 9.—CHARACTERISTICS OF STORAGE RESERVOIRS

Item No.	Name of project	Stream	Type	Depth, in feet	CAPACITY, IN ACRE-FEET		AREA, IN ACRES				VALUES OF m		VALUES OF C_w		From depths
					Computed	By diagram*	Computed	By diagram*	Average depth, in feet	Ratio: Mean depth / Maximum depth	Computed	By diagram	Computed	By diagram	
1	Bronte	Colorado River	Gorge	25	2 986	.....	330	.....	.....	0.267	3.65	3.75	0.02033	0.015	.....
				50	27 202	22 500	1 985	1 600	13.70		4.08	3.90	0.00353	0.006	50–75
				75	131 887	105 000	7 168	5 400	18.40		3.74	4.15	0.01343	0.002	50–100
				100	405 652	300 000	15 175	11 500	26.73		3.25	3.39	0.2098	0.15	75–100
				25	4 660	.....	481	.....	.....		3.15	3.35	0.3294	0.15	50–100
				50	60 778	80 000	3 953	4 500	15.38		3.02	3.30	0.5994	0.20	50–115
2	San Angelo	Concho River	Hill	75	258 176	275 000	10 857	10 300	23.78	0.334	3.00	3.30	0.6572	0.25	75–100
				100	657 198	610 000	19 830	18 200	33.14		.....	.....	.....	.....	75–115
				115	1 020 357	1 000 000	26 585	24 000	38.38		.....	.....	.....	.....	.....
			Hill	50	25 090	28 000	1 447	1 250	17.33		2.69	2.7	0.2539	0.7	50–90
			Hill	90	124 990	120 000	3 647	3 000	34.27		2.63	2.6	0.8408	1.0	50–150
3	Junction	South Llano	Flood-plain	150	430 850	420 000	6 579	6 200	65.49	0.405	2.29	2.5	4.024	2.0	50–190
			Flood-plain	190	754 130	775 000	9 829	8 800	76.88		2.47	2.47	1.757	2.5	90–150
			.....	.....	.....	.....	.....	.....	.....		.....	2.35	.....	3.5	90–190
			Gorge	25	1 130	.....	140	.....	.....		.....	.....	.....	.....	.....
			Gorge	50	16 159	34 000	1 554	2 100	10.40		4.81	4.18	0.000069	0.0015	50–100
4	Breckenridge	Clear Fork, Brazos	Hill	75	103 567	130 000	5 331	5 100	19.43	0.311	3.86	4.05	0.005518	0.002	50–110
			Hill	100	289 594	320 000	9 749	10 000	29.70		3.27	3.50	0.0527	0.03	75–100
			.....	110	395 524	430 000	11 565	12 500	34.20		3.22	3.50	0.1051	0.03	75–110

*By author's Figs. 4 and 6, using values of m and C_w computed by using maximum depth shown in this table for the particular site.

TABLE 9.—(*Continued*)

Item No.	Name of project	Stream	Type	Capacity, in Acre-Feet: Depth, in feet	Capacity, in Acre-Feet: Computed	Capacity, in Acre-Feet: By diagram*	Area, in Acres: Computed	Area, in Acres: By diagram*	Area, in Acres: Average depth, in feet	Area, in Acres: Ratio: Mean depth / Maximum depth	Values of m: Computed	Values of m: By diagram	Values of C_w: Computed	Values of C_w: By diagram	From depths
5	Palo Pinto County	Brazos	Hill	40	27 970	25 000	1 921	1 975	14.56		2.75	3.58	2.355	0.060	40–100
			Hill	75	220 260	220 000	8 088	9 700	27.23		2.75	3.39	2.355	0.105	40–140
			Hill	100	744 816	600 000	19 206	19 200	38.78		2.58	3.39	5.153	0.105	40–180
			Hill	140	2 031 900	2 000 000	47 500	45 000	42.78	0.336	3.27	4.41	0.2148	0.0015	75–100
			Gorge	180	4 753 070	5 000 000	89 910	86 000	50.86		3.54	3.58	0.06195	0.05	75–140
			.....	.....	.....	.....	.....	.....	.....		....	2.82	.....	1.5	100–140
			.....	.....	.....	.....	.....	.....	.....		....	3.00	.....	0.70	100–180
6	Jonah	San Gabriel	Hill	50	18 100	13 000	1 175	800	15.40		3.25	.....	0.6167	.....	.....
			Hill	70	56 880	41 000	2 703	1 900	21.07		3.32	3.35	0.04467	0.045	50–90
			Hill	90	133 280	100 000	4 937	3 500	27.00	0.282	3.33	3.40	0.04169	0.04	50–100
			Hill	110	267 680	200 000	8 503	5 700	31.48		3.49	3.40	0.02042	0.035	50–130
			Gorge	130	485 100	360 000	13 247	8 800	36.62		3.54	3.55	0.01672	0.02	90–130
7	Forth Worth	Clear Fork, Trinity	Gorge	40	6 704	12 700	813	1 050	8.25		4.85	3.9	0.000242	0.0035	40–80
			Hill	80	96 786	125 000	4 049	5 200	23.90		3.35	3.75	0.02422	0.006	40–100
			Hill	100	206 934	260 000	7 195	8 600	28.76		3.48	3.70	0.02269	0.007	40–120
			Hill	120	382 380	460 000	10 335	13 000	37.00	0.308	3.24	3.20	0.08628	0.09	80–100
			.....	.....	.....	.....	.....	.....	.....		....	3.35	.....	0.045	80–120
			.....	.....	.....	.....	.....	.....	.....		....	3.55	.....	0.02	100–120
8	Bee Mountain	Brazos	Gorge	50	15 860	11 000	929	810	.....	.....	.....	3.8	.....	0.005	50–100
				100	228 170	215 000	10 498	7 700	21.73	0.217	4.60	4.0	0.0005731	0.002	150–120
				120	519 250	480 000	18 610	14 000	27.90	0.249	4.30	4.5	.....	0.0003	100–120
9	Brazos—Madison County	Navasota	Hill	20	63 485	72 000	7 971	4 300	.....	.....	.....	2.25	.....	70.0	20–30
				30	165 380	182 000	12 408	6 900	.....	.....	.....	2.30	.....	60.0	20–40
				40	318 660	350 000	18 248	9 500	17.46	0.436	2.29	2.40	71.02	50.0	30–40

* By author's Figs. 4 and 6, using values of m and C_w computed by using maximum depth shown in this table for the particular site.

Mr. Sutherland's method of studying the characteristics of storage reservoirs has been applied to thirty-four reservoir areas in Texas that have been surveyed by the Topographic Branch of the U. S. Geological Survey, co-operating with the Texas Board of Water Engineers. Some of the areas studied are mapped on a scale of 1 : 24 000, with a contour interval of 5 ft.; others are mapped on a scale of 1 : 48 000, with contour intervals of 10 and 20 ft. At some of these areas two or more locations for the dam were considered, making a total of fifty-five reservoir sites studied. The development of some of them would be impractical; only a few of them have been, or are being, developed. They merely represent some of the investigations that have been made by the Texas Board of Water Engineers, looking toward the development of the water resources of the State. In this investigation, some of the dams were carried to maximum height for the purpose of applying the author's method of study, and not as an indication of what might be practical in the economic development of these reservoirs.

The capacity curves of the reservoirs in Table 9 were drawn as straight lines on logarithmic paper, as shown by the author. For most of these reservoirs the plotted points showing capacity, fit closely the lines thus drawn. For some the plotted points give a uniform upward curve; while others give a uniform downward curve. Some of them give a decided curve at the lower heights, but soon develop into a tangent as the height increases. The extension of some of these lines to the 100-ft. height, in order to calculate the constants in the author's formula, give storage values that are probably not reliable and would affect the accuracy of the values of the constants, C_w, calculated therefrom.

In the arbitrary classification of the reservoirs into four types, the particular class into which a reservoir may fall depends upon the shape of the cross-section of its area, and somewhat upon the height of the dam considered, as the value of m may be varied considerably by changing the height of the dam. This classification may have little to do with the general topography of the surrounding country outside the stream channel. However, with the foregoing observations in mind, the classification as made and used by the author appears very satisfactory and should be useful.

Table 9 shows the characteristics of seven of the fifty-five projects studied. The elevation of the bottom of the reservoir is given for each point so that comparison may be made of the values of m in reference to elevation on any particular stream. The computed values of the constants, m and C_w, for different heights of dam, are given in order that comparison may be made. In order to compute the value of C_w in Item No. 9 (Table 9) by Equation (3), it was necessary to extend the height-capacity curve from Height 40 to Height 100. This may have given a capacity for 100-ft. depth that would be considerably in error, which, in turn, would cause errors in the computed value of C_w and also in the capacities and areas taken from the diagrams.

The author's reservoir capacity diagram, Fig. 4, as well as the other alignment charts are very ingenious, and should often be found useful in the preliminary examination of reservoirs. Comparing the computed values of m and C_w with values taken from Fig. 4, by using intersecting lines from two known

depths and capacities, quite a difference in values is often noted. Where such differences are found, capacities taken from the diagram for other depths may be considerably in error, as may be seen from Table 9.

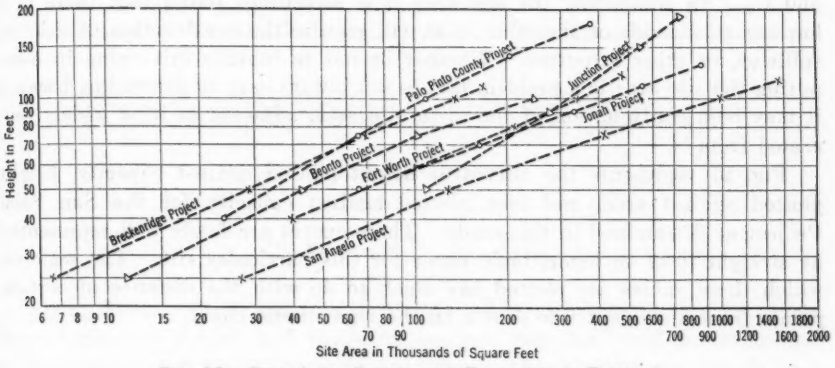

FIG. 26.—SITE AREA CURVES FOR PROJECTS IN TABLE 9.

Computing the value of the ratio, the average or mean depth divided by the maximum depth, to any given elevation for the fifty-five sites considered, the mean value is 0.3148, corresponding to $m = 3.18$. Most of the reservoirs fell into the "hill type" classification, which probably accounts for the smaller variation in the values of m.

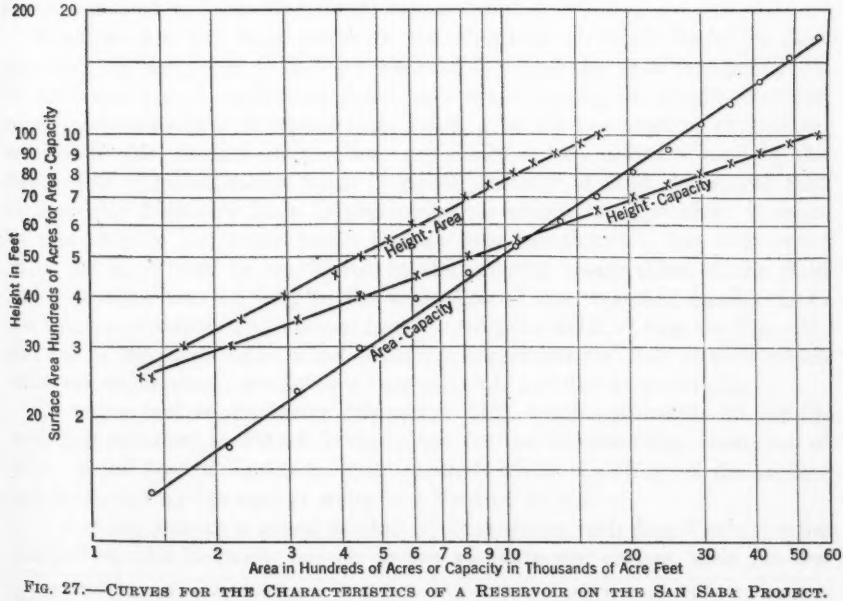
Height in Feet
Surface Area Hundreds of Acres for Area-Capacity
Area in Hundreds of Acres or Capacity in Thousands of Acre Feet
Height-Area
Height-Capacity
Area-Capacity

FIG. 27.—CURVES FOR THE CHARACTERISTICS OF A RESERVOIR ON THE SAN SABA PROJECT.

Areas were computed for the sites in Table 9 and for different depths. These are plotted in Fig. 26. The plotted points have been connected by

dotted lines for easier identification, no attempt being made to fit a straight line to them. Table 9 also shows a comparison of the computed values with the values taken from the author's diagrams for capacity, surface area, m, and C_w. In computing the site area it is sometimes found that there is a low sag at one side or the other, that will require the construction of a levee, spillway, or other structure. Whether or not to include this value in computing the site area is a problem that is not always easy to determine, because it may be a continuation of the main site area, adjacent to it or widely separated from it.

Fig. 27 represents the curves of height plotted against capacity, height plotted against area, and area plotted against capacity for the San Saba Project as determined in this study. These curves are fairly well represented by straight lines on logarithmic paper for this particular site. The scale on which these values are plotted has much to do with the distance of certain points from a straight line drawn on the logarithmic chart.

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DISCUSSIONS

THEORY OF SIMILARITY AND MODELS

Discussion

BY ED S. SMITH, JR., ESQ.

ED S. SMITH, JR.,[17] ESQ. (by letter).[17a]—The author's paper attacks the present usage of models and unjustifiably urges that the broad generalization of Newton be used for all cases of fluid flow. This viewpoint seems to be a mistaken one since the correct and adequate basis for flow similarity in pipes is the Reynolds number and not that of Newton. The writer's experience with quantity-rate fluid meters shows Mr. Groat's theory to lead to an incorrect basis for flow similarity for pipes and confirms the present usage, based on the Reynolds number, which permits accuracies of within one-fourth of 1% to be realized with Venturi meters when metering one fluid-using datum obtained with another fluid, for example.

Excellent use has been made by Mr. Groat of Newton's basis for flow similarity in its proper field—open channel flow and the fluid transportation of sand and gravel (non-fixed shape and non-homogeneous fluid)—but the writer maintains that Newton's basis (with g as the acceleration of gravity, as used by Mr. Groat) simply does not apply to the ordinary case of the steady flow of homogeneous fluids in closed conduits of fixed shape, and that consequently Newton's basis is absolutely not generally applicable; it must be used only in its proper field. For the case mentioned it can only enter in so far as it may be considered in the internal acceleration of the fluid which is taken care of fully in the derivation of the Reynolds number from the balance of inertia and viscous forces within the fluid. Thus, the Reynolds number is the basis upon which data are correlated for this case in which whatever accelerations are involved are certainly not due to gravitation.

Newton's law of similarity (Equation (1)) applies primarily to liquids with free surfaces, as Froude brought out for the wave-making resistance of ships, or the flow of liquids in open channels where the effect of the surface enters—including the case of weirs and Venturi flumes.

For steady flow in a closed conduit of fixed shape, including Venturi tubes and orifices, the Reynolds number is the adequate and correct basis for flow

NOTE.—The paper by Benjamin F. Groat, M. Am. Soc. C. E., was published in October, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: January, 1931, by Messrs. H. Bateman and W. P. Roop; and February, 1931, by Messrs. A. V. Karpov and Jasper O. Draffin.

[17] Hydr. Engr., Builders Iron Foundry, Providence, R. I.

[17a] Received by the Secretary, January 2, 1931.

similarity and the correlation of data, as established beyond question by innumerable experiments. This criterion also holds for airplanes, dirigibles, submarines, and for air and water propellers, with an additional term to take care of the various slip ratios.

Fortunately, it is possible to subject fluid flow phenomena to an independent analysis which definitely establishes the variables and operators involved so that one may test the fitness of a generality, such as Newton's expression, to the case of fluid flow through a closed conduit of fixed shape. For this case the Reynolds number is found to be pertinent. In Fig. 2, the flow, Q, through a closed conduit is divided into two streams. One stream flows through a long capillary tube, M, in which the resistance to flow is due to viscous forces, and the other flows through a nozzle, N, or orifice, in which the resistance to flow is due to the inertia of the fluid.

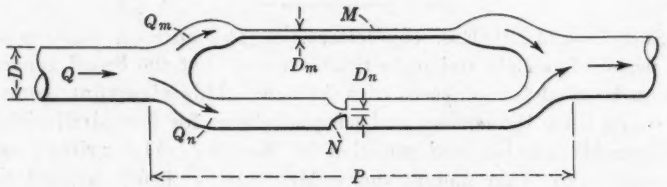

FIG. 2.

In another paper[18] the writer has demonstrated that, from the known laws of flow, the ratio of flows (that is, "flow similarity") and the coefficient for the example given, depend only on the Reynolds number and not upon Froude's ratio (Newton's law involving the acceleration of gravity), as specified by Mr. Groat. Consequently, for closed conduits, gravity is shown to have no effect on the flow distribution. A familiar proof of this is that the direction of the axis of flow in a nozzle with respect to the vertical has no effect upon the performance of the nozzle. The Reynolds number is adequate and proper for use in the example cited herein and does not require the assistance of Newton's ratio for correlating data. The writer would appreciate Mr. Groat's confirmation, as an authority, that this is correct or, if he should fail to agree, then to demonstrate a solution of this problem in his closing discussion.

The method of similarity is a most valuable one in the field of fluid flow, but it must not be used blindly. In every case, the engineer must consider carefully which of the various physical quantities and phenomena is involved, and then select the proper dimensionless ratio as the basis for similarity. In the case of fluid flow in closed conduits this is the Reynolds number; for open channels subject to wave action it is Froude's ratio (Newton's ratio, using g as the acceleration of gravity); while, for expansible fluids, it is the acoustic velocity ratio. For example, in the case of airplane propellers (of which Mr. Groat treats in Appendix I) the tips frequently approach the velocity of sound and a correction for the compressibility of air is certainly required. The writer has proposed the use of the acoustic ratio, $\frac{x}{k}$, in which,

[18] "Quantity-Rate Fluid Meters," by Ed S. Smith, Jr., *Transactions*, Am. Soc. Mech. Engrs., Paper HYD-52-7*b* (Fig. 6).

x is the ratio of differential to absolute pressure and k is the ratio of specific heats, for fluid flow through quantity-rate meters. This acoustic ratio, $\frac{x}{k}$, may readily be modified so as to be applicable to the air-propeller problem when testing with another gas than air; this may prove to be more convenient in actual use than the acoustic velocity ratio.[19]

Mr. Groat's treatment of the case of air propellers (which does not involve Newton's, or Froude's, ratio) is inadequate and incorrect. It emphasizes the point that the various engineering fields require engineers actively engaged therein to be familiar with the use of the method of similarity so that they can make effective use of it. This is preferable to having "similarity engineers" attempting to use the method in fields with which they are unfamiliar.

While the method of similarity has been highly useful in the field of fluid flow, this has been so largely because the pertinent variables have been sufficiently simple (density and viscosity) to be easily susceptible to attack by the method of dimensional analysis.

However, for structural similarity, the dimensional method may easily require considerable assistance before the results will be most useful, since hardness, toughness, resistance to shear, tearing, and plastic deformation seem scarcely to have been defined adequately in a dimensionally sound way. Furthermore, these bring in the factor of direction so that the problem introduces vector quantities which may or may not be covered by dimensional analysis in determining the proper basis for similarity—along this line is the familiar fact that work and torque have the same dimensional units: $F \times L$.

Complete use also must be made of established methods of detailed analysis wherever possible for checking the application of this general method of similarity to structures, or any new field. Otherwise, it will suffer temporary loss of confidence and, therefore, usefulness, due to improper use without adequate physical understanding. Wonderfully useful as this method is, it does not enable one to obtain something for nothing, being in fact a convenient basis on which to correlate data for most effective use.

The conclusions from the foregoing comments are:

1.—The present usage of models for flow in pipes, based on the Reynolds number without the encumbrance of Newton's ratio (actually Froude's), is entirely consistent with sound theory, being unquestionably established by a great mass of accurate data.

2.—For this case of flow in pipes, it is not necessary to synthesize fluids (to meet the joint requirements of the Reynolds number and Newton's ratio, since the latter does not apply).

3.—It is necessary to follow the specific dimensionless ratio which is pertinent to any given case, rather than to attempt to use a too broad (and, hence, generally incorrect) generalization, such as Newton's ratio, for all cases. It is a step backward to apply a universal cure-all to cases in which it obviously does not work.

These conclusions are completely in opposition to those of Mr. Groat.

[19] See "Similarity and Model Experiments," by A. H. Gibson, *Engineering*, March 21, 1924, p. 358, for correct "air screw" theory.

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DISCUSSIONS

THE BUTTRESSED DAM OF UNIFORM STRENGTH

Discussion

BY A. A. EREMIN, ASSOC. M. AM. SOC. C. E.

A. A. EREMIN,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—The author has developed a new method of designing buttresses in which first principal stress compression is uniform and second principal stress is zero.

The present method of computing principal stresses is based on assumptions which have numerous limitations. The lines of uniform principal stresses in a buttress vary, due to changes in temperature, shrinkage of concrete, and lateral stiffness of the buttress. Professor Otto Mohr[12] has stated that in ten dams the stresses will be different regardless of the fact that they may be constructed under similar conditions.

Principal stresses in buttresses are commonly computed with the assumption that the normal stresses in the transverse section vary linearly. Therefore, the deformations in a buttress must obey the principle expressed by Navier, that transverse plane sections will remain plane after loading and in Hooke's law, that strain is proportioned to stress or that the modulus of elasticity of material is constant at all stresses.

The modulus of elasticity of concrete is variable. However, in sections with all compressive stresses, the error from using a constant modulus of elasticity is negligible, as was also stated by the writer in another discussion.[13]

A number of experimental tests with model walls was made in order to prove Navier's principle in the transverse sections of concrete gravity walls or buttresses. Maurice Lévy[14] investigated a wedge-shaped wall with water pressure on one side, and stated that the measured deformations are in close agreement with the Navier principle. However, a small deviation was noticed when a vertical load was applied at the crown of the wall. Later, Mr. J. H. Mitchell and Dr. Fillunger[14] confirmed the statement made by Lévy, and, furthermore, they found a departure from the Navier principle in the lower

NOTE.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in November, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: January, 1931, by Messrs. James Girand and Fred A. Noetzli; and February, 1931, by Messrs. B. F. Jakobsen and Calvin V. Davis.

[11] Asst. Designing Engr., Bridge Dept., State Highway Comm., Sacramento, Calif.

[11a] Received by the Secretary, December 22, 1930.

[12] "Die Staumauern", by Dr. N. Kelen, p. 85.

[13] *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1437.

[14] "Die Staumauern", by Dr. N. Kelen.

sections of a model wall (Fig. 24), if a horizontal force or a moment is applied at the crest. Karl Wolf[14] has stated that Lévy's conclusions are correct for walls well keyed into solid rock foundations. For his tests he used a model of a wall with its vertical face on the water side and the down-stream side inclined at 45°, and his investigations are based on St. Venant's theory.

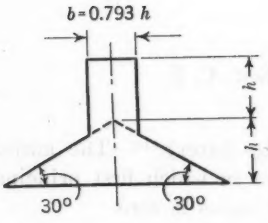

FIG. 24.

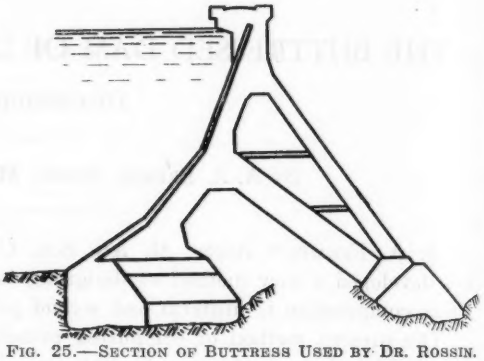

FIG. 25.—SECTION OF BUTTRESS USED BY DR. ROSSIN.

Experimental tests have proved also that shear stress in a transverse section of a buttress does not follow a linear distribution.[14] Thus far, however, all experimental tests to determine the distribution of stresses in gravity dams or buttresses have been performed with small-sized models and their results may be in error when applied to the massive sections used at the present time. Involved mathematical analyses of buttress sections based on an uncertain distribution of stresses may be worthless unless a more precise method of stress computation is discovered.

The author has properly indicated the advantages of the buttress composed of the elementary arched columns; however, the following factors should be considered in the design of the arched columns:

1.—If the deck is monolithic and separated from the arched columns by a frictionless joint, as shown in Figs. 4 and 5, secondary stresses in the deck will occur due to unequal settlement of the columns or to a variable change in the temperature of the buttress.

2.—A rock foundation is seldom homogeneous or isotropic and, therefore, a deformation of the foundation should be considered in any precise design of buttresses.

3.—If the elementary deck strips are monolithic with the arched columns there will be reaction forces along the center line of the deck strips where these strips touch each other (Figs. 5 and 6). These forces will vary with the swelling effect of the deck due to the moisture in the concrete, and with the temperature variation in the deck, and the result will be bending stresses in the arch columns, and eccentricity of pressures in the foundations.

The idea of a unit system of buttresses was used by Dr. Rossin[15] (see Fig. 25). This buttress is more economical than a monolithic buttress and the

[15] "Die Staumauern", by Dr. N. Kelen, p. 264.

stresses in the members of the frame may be determined with reasonable precision by the method of least work or any other design method applied to the rigid frame.

The height of Dr. Rossin's buttress may be increased by using the hollow type of columns or by columns stiffened with counterforts or struts.

For the advancement of knowledge concerning buttress design, experimental tests on stress distribution on a large scale are needed. This paper is a meritorious contribution to the advancement of the theory, and the application of the author's new design methods to actual practice is highly desirable.

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DISCUSSIONS

ECONOMICAL DESIGN OF HYDRAULIC PIPE-LINE DREDGE

Discussion

BY ARTHUR L. COLLINS, ASSOC. M. AM. SOC. C. E.

ARTHUR L. COLLINS,[2] ASSOC. M. AM. SOC. C. E. (by letter).[2a]—In changing from the steam-operated pipe-line dredge to the motor-driven units, it is well to bear in mind the peculiar characteristics of the centrifugal pump.

In present-day steam-dredge operation the efficiency of the pump is given little consideration. It is noted that in determining the power requirements for the dredge, *New Jersey*, the pump efficiency was assumed to be 70 per cent. While pumps with an efficiency as high as 90% can now be obtained for clear water, it would appear to the uninitiated that 70% could be depended upon for the dredge. However, when the dredge pump is properly tested. it is generally found that the efficiency is somewhere between 55 and 65 per cent. This is about the minimum at which a centrifugal pump can be expected to operate.

In passing from the steam-driven to the motor-driven pump, whether the latter receives its electrical energy from a power station or from Diesel engines, it is of great importance that the temperamental operating characteristics of the pump be thoroughly understood.

Fig. 4 represents the characteristics of a typical dredge pump driven by electric motor. Only the fundamental information regarding the test is shown. In the motor-driven pump it is assumed that the speed is practically constant. This is the point that accounts for the condition of undesired instability, as indicated by the test. For example, the curve of total lift and capacity is very flat. At 24 000 gal. per min., the head is 131 ft. At 18 000 gal. per min., the head is 138 ft. In other words, the pump capacity falls 25% in a change of 7 ft. The power input to the motor falls from 1 060 to 880, or 180 kw. In normal operation the suction lift on the pump will fluctuate more than the specified 7 ft. from minute to minute, depending upon the material which is picked up by the pump. The fluctuation is 7 ft., more or

NOTE.—The paper by John Francis Cushing, M. Am. Soc. C. E., was published in November, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: February, 1931, by R. L. Vaughn, Assoc. M. Am. Soc. C. E.

[2] Cons. Engr., Berkeley, Calif.

[2a] Received by the Secretary, February 2, 1931.

less, and, consequently, the pump is in a continuous unstable condition, regarding capacity and power.

Ordinarily, there is a recording suction gauge installed on the pump. This gauge should show considerable variation, but this effect should not be instantly transmitted to the pressure gauge in a well-designed pump. A widely fluctuating pressure gauge is an indication of the sensitive pump.

A steam-operated pump, with its more or less flexible control, has a tendency to ease the characteristics of the pump. There is a balance between power and speed, as well as between head and capacity. The electrically-driven pump cannot slow down, but must maintain a nearly constant speed. The motor-driven pump is likely to be noisy, due, in part, to the constantly changing internal flow velocities.

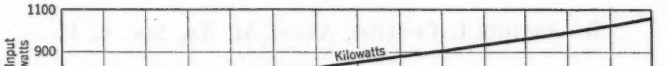

FIG. 4.—CHARACTERISTICS OF A DREDGE PUMP.

The variable-speed motor does not eliminate the objectionable features of the constant-speed motor. The motor is operating at reduced efficiency when on any contact less than full speed. While on this contact, it is not far from being a constant speed motor, the same as when operating at full speed.

In passing from the steam-engine drive to the electric drive, the engineer must take into consideration the difference between a constant-speed and the variable-speed, steam engine. Of course, the steam engine will be controlled within certain limits by the governor, but the governor can be set at any desired speed.

There are centrifugal pumps now in various parts of the United States which were installed in the late Seventies, and afterward, and which were operated by steam. Strange to say, some of these pumps have a very good efficiency. They are efficient because they can be driven at a selected speed. Try to drive them electrically with a constant speed, and it will be found that they cannot be satisfactorily operated, due to the conditions shown in Fig. 4.

It can be said that probably the pump given as an example did have an efficiency of 70% when it was first installed. Once a dredge pump is in operation, it undergoes a series of alterations. Parts of the pump interior are chipped intentionally or worn by service. Patterns for new parts are altered

and the pump finally emerges as a highly temperamental piece of equipment. In this case it has lost its stability and is running wild as to power and capacity, and is only kept within bounds by reason of motor speed. It will be merely a coincidence if the pump will test 70% at any period after it has started operation.

More attention must be given to the design of the dredge pump, not only to save money in power consumption, but to eliminate wear and repairs. During the past few years much attention has been given to the design of the sewage pump. The pump companies have had to design the pumps for motor drives and they have developed pumps with characteristics that should be copied by the dredge designer. By making use of the similarity of the sewage pump, it is possible to build the larger sizes as used on dredges, and anticipate the desired characteristics. The electric drive will require a properly designed pump.

The writer does not consider the injection of salt or color into a line as the best method for obtaining data which will enable one properly to analyze the operations of the pump. The most satisfactory method is first to test the pump with clean water. The curve similar to that in Fig. 4 is obtained by pumping against a partly closed valve, and noting the action of suction and pressure gauge. The Pitot tube is used in the suction pipe for the capacity measurements. This method provides the true operating characteristics of the pump which are comparable from time to time.

From the viewpoint of the engineer endeavoring to check the characteristics, the color or salt-water injection method is not to be preferred. Its only justification would be to check or calibrate the gauges under load conditions after the fundamental information is obtained. The fundamental information is important, because one can check the condition of the pump within, say, 2%, from time to time, and can do this in a few minutes and with no loss of dredge time.

No doubt there are many dredge owners with steam-driven pumps who have their eye on the motor drive. It would be disastrous to assume the pump efficiency as 70 per cent. In view of the uncertainty of the operating characteristics of the pump unit, it is advisable to base the power requirements on a pump of low efficiency.

Too much stress cannot be placed upon the importance of knowing the exact performance of the pump. In operating a pump direct by a Diesel engine, the owner is very likely to be gravely disappointed, if the pump efficiency is rated too high. To begin with, the gas engine or Diesel engine is not generally given an overload rating. An electric motor with plenty of reserve power from the line can readily provide an overload of 25 per cent. The Diesel engine must be liberally rated, otherwise the centrifugal pump may be found to require more power than was anticipated by the assumed efficiency.

Generally speaking, little or no attention is paid to the matter of efficiency in the dredge pump after it is once in operation. The characteristics of the pump can change within wide limits, and the pump under the new conditions will require more power and more repairs. Without a systematic plan for

checking the condition of the pump every few weeks, there is no clue as to how it can be improved. There may have been some justification for the lack of incentive on the steam-driven dredge to make frequent tests, due to the flexibility in speed and power of the prime mover.

With the advent of the motor drive, generally little or no information of importance is available in the files of the engineering department of the dredge owner to enable him to anticipate accurately the performance of the same pump under the motor drive. To operate a motor-driven dredge which will show good efficiency over a long period of time, will require that a constant check be kept on the characteristics of the pump.

It will require the concentrated effort of a technical engineer who has a full working knowledge of the meaning of the characteristic curve of the pump. The dredge captain should not step from the steam dredge to the motor-driven dredge without recognizing the value of the technical engineer in analyzing the pump problem. The cost of handling the material is not only affected by an excess cost of power, but also by repairs, shut-downs, and improper pipe-line velocities, all contingent upon the design of the pump; and which design can be altered quite unintentionally.

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DISCUSSIONS

FLOW OF WATER IN TIDAL CANALS

Discussion

BY MESSRS. G. T. RUDE, EUGENE E. HALMOS, AND W. M. BLACK

G. T. RUDE,[24] M. AM. SOC. C. E. (by letter).[24a]—In the closing discussion of the paper, "Tides and Their Engineering Aspects",[25] the writer made the statement that Colonel Brown had had charge of the lowering of the Chesapeake and Delaware Canal to sea level, and that any studies he had made of wave propagation in the canal would be awaited with interest. The Engineering Profession is indeed indebted to Colonel Brown for this contribution to the subject of restricted and complex tidal flow.

The writer desires to preface this discussion with the explanation that, while exception is taken to certain statements regarding ocean currents and the dominant tides of the oceans, which are considered not in entire accord with these phenomena as accepted by tidal students, it is obvious that they are used by Colonel Brown merely to round out his paper. In no way do they detract from the value of the study as to the flow of water in tidal canals, assuming for the moment that the writer's statements as to these phenomena are the correct ones.

Undoubtedly, from his exhaustive study, Colonel Brown can easily and successfully clarify one phase of his "reflected wave" theory. A visualization of a reflected wave, in the case of a canal with a dead end, is not difficult, nor is its agency in consuming the unexpended energy of the wave upon arrival at the end. In the case of the canal with an outlet into a body of water, however, it would appear that the propagation of the wave would continue out into the outlet body in its original form and be dissipated in that body instead of being propagated back into the canal as a "reflected wave".

In the definitions, Colonel Brown uses the term, "strength of the tide", apparently with reference to the current. It is suggested, for uniformity in the nomenclature of this particular subject, that the term "current", be applied to the horizontal movement of the water and the term "tide", to the periodic

NOTE.—The paper by Earl I. Brown, M. Am. Soc. C. E., was published in December, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: February, 1931, by R. L. Faris, M. Am. Soc. C. E.

[24] Commander, U. S. Coast and Geodetic Survey, Washington, D. C.

[24a] Received by the Secretary, January 31, 1931.

[25] *Transactions*, Am. Soc. C. E., Vol 92 (1928), p. 711.

vertical movement. The term, "tide", also appears to be limited to the motion of a "derived" wave.

"Mean canal level" or "mean river level" is taken as the mean of the high and low-water levels at any point. It is customary, however, among tidal mathematicians to differentiate between "mean sea level", "mean river level", or "mean canal" and "mean tide level". The term, "mean tide level", is applied to the plane exactly half-way between the high and low waters and is obtained by taking one-half the sum of all high waters and low waters. Mean sea level, however, is obtained by integrating the tide curve, at least to the extent of taking the average height of the curve at frequent intervals. It is customary to use the hourly heights for this purpose. Mean river level or mean canal level is the mean height of the water surface as influenced by the tide, freshets, and bottom slope, and is obtained in the same manner as mean sea level.

The difference between mean tide level and mean sea level, however, is small, and if the series of observations is short, the two planes may be considered for practical purposes as coincident. On the Atlantic Coast the plane of mean sea level lies above the plane of mean tide level—at Portland, Me., 0.03 ft.; at New York, N. Y., 0.05 ft.; at Atlantic City, N. J., 0.03 ft.; and at Fernandina, Fla., 0.10 ft. On the Pacific Coast mean sea level is the lower of the two—at Seattle, Wash., 0.01 ft.; at San Francisco, Calif., 0.06 ft.; and at San Diego, Calif., 0.05 ft. This difference in elevation of the two planes is due to the fact that the tide curve is not a simple sine curve; actually, the larger part of the curve may be above or below the mean level of the water as determined from the average of the high and low waters.

Apparently, the hydraulic characteristics in the canal are not taken into consideration. The tidal data for the Chesapeake and Delaware Canal, furnished the U. S. Coast and Geodetic Survey by the Army Engineer Corps, appear to indicate that the current is primarily hydraulic in the western portion of the canal, although it loses this characteristic at the Delaware Bay end. The present current predictions made by the Coast and Geodetic Survey in the Atlantic Coast Current Tables are for the location of the railroad bridge. These predictions are based on the assumption that the current is hydraulic at that point.

Under the paragraph, "Tidal Waves in the Sea", Colonel Brown states that tidal currents in shallow waters near the shore line are mass currents acting for the entire depth of the sea. Observations indicate that, in rivers and canals, the maximum velocity is just below the surface. Mr. Parsons calls attention to this fact in his paper on the Cape Cod Canal.[26] Current observations made by the U. S. Coast and Geodetic Survey have shown varying results in short series. The observations, however, over a longer series substantiate Mr. Parsons' statement. At the principal station of a section of the Boston Harbor Current Survey by the Coast and Geodetic Survey, near Deer Island Light, where fourteen days of observations were obtained, the following velocities were observed at flood strength: 7-ft. depth, 3.78

[26] "The Cape Cod Canal," by William Barclay Parsons, Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1.

knots; 13-ft. depth, 3.81 knots; 32-ft. depth, 3.77 knots; and 51-ft. depth, 3.75 knots. The corresponding directions were 267°, 270°, 271°, and 268°, showing little variation in direction.

In the open ocean, however, where the terrain has little or no influence, the currents act with ever-decreasing velocity with the increase in depth and, too, the direction changes constantly as the depth increases, clockwise in the Northern Hemisphere and counter-clockwise in the Southern Hemisphere.

In the paragraph mentioned, Colonel Brown also states that:

"The currents [in the open sea] sometimes become gyratory because of the combination of longitudinal currents with lateral currents which come and go, corresponding to the alternate rise and fall of the water on shore, or of the crossing of two co-existent waves, the directions of which intersect at an angle."

The work of an internationally known mathematician, corroborated by actual observations of ocean currents, demonstrates that this gyratory or rotary movement is a distinct and ever-present characteristic of these currents, not only along the coasts, but over the entire ocean. These phenomena were first suspected by Fridtjof Nansen in his historic drift in the Arctic ice pack during 1893-1896.

Nansen's deductions were reported to Helland-Hansen, who demonstrated mathematically that this rotary movement is due to the deflective force of the earth's rotation, the direction of rotation of the currents being clockwise in the Northern Hemisphere and counter-clockwise in the Southern Hemisphere. Nansen's theory of this rotary movement has been proved as an ever-present characteristic of ocean currents, not only by the mathematics of Helland-Hansen, but also by actual current observations made since 1915 by the U. S. Coast and Geodetic Survey on every light-vessel along both coasts of the United States. It has also been checked in the open ocean by Helland-Hansen, utilizing innumerable reports of current effect upon ocean-going vessels.

These ocean currents are never slack;[27] their direction changing continually in a rotary movement at an average rate of about 30° per hour for semi-diurnal currents. At the station of observation the velocities vary throughout a tidal cycle and the directions and velocities at a station, plotted to scale, roughly approximate an ellipse.

It appears that Colonel Brown has not accepted the "stationary wave theory" of Harris as an explanation of the changed regimen of the tide in certain bodies of water in which the tide occurs almost simultaneously throughout the entire length of the body and in which the turn of the current occurs near the time of high and low water instead of at the time of half-tide. Under the heading, "Tidal Waves in the Sea", Colonel Brown states:

"In general, near the coast line, the situation is intermediate between the state of free propagation of the tidal wave, and that of filling a basin of small dimensions; the regimen of the tide becomes changed. The reversal or turn of the tide [current] is then no longer effected at mid-tide, but approaches

[27] "Tides and Their Engineering Aspects," *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 656.

closer to the times of high and low water, and the maximum velocity approaches mid-tide. The velocities then differ from their theoretical value and, in order to interpret the phenomena observed, one is tempted at times to have recourse to considerations based upon the hydraulics of running water and the influence of surface slope. Under these conditions the tidal wave loses its usual characteristics, entirely or in part, and in order to account for observed facts, it must be considered as formed by a series of small waves of translation each of which is propagated according to its own proper law."

Harris' stationary wave theory accounts for these and other apparent anomalies in the tides in a more rational manner than the older theories and it is being accepted both in the United States and abroad. This theory would explain the dominant tides of the world as the results of oscillations or stationary waves, set up and maintained by the periodic tidal forces of moon and sun. In most inland bodies of water connected with the ocean the tide is due to a progressive wave thrown out by a near-by stationary wave in the ocean. In a body of water, however, of a size and depth to maintain an independent stationary wave, the tides may be caused by its own stationary wave.

It could be shown that Harris' stationary wave theory[28] furnishes a logical explanation of the tidal characteristics of some regions, which differ from the usual tidal regimen, such as the reversal of the current at the time of high and low water instead of at the time of mid-tide, mentioned in Colonel Brown's paper; the almost simultaneous occurrence of high or low water throughout the length of the body; the increase of the range toward the head of the body, represented in the large range at the head of the Bay of Fundy; and the strictly solar tide of the Island of Tahiti, with its high and low waters occurring at about noon and midnight, and about six o'clock in the morning and evening.

Under the heading, "Canal Connecting Two Bodies of Water Each Having an Independent Tide of Its Own", Colonel Brown accepts the theory advanced in the past by some engineers, that the characteristics of the tidal movement in such canals, for example, Seymour Narrows, Deception Pass, East River, etc., are due to the interference of two opposed systems of waves, each independent of the other. The evidence, however, from direct tidal and current observation, supported by the accuracy of predictions of tides and currents based on hydraulic considerations, indicates that the movement of the water in such straits is primarily hydraulic, brought about by the difference of the tidal characteristics in the connected bodies of water and the differences in the time and height of the tide in the two bodies.[29]

The writer would like to inquire of Colonel Brown whether the values in the columns headed "Bourdelles" and "Lévy" in Tables 3, 4, and 5, are observations made by Bourdelles and Lévy with which the computed values in the column, "Present Theory" are compared, or whether they are values computed by Bourdelles and Lévy compared with computed values by the "Present Theory"?

[28] Manual of Tides, Pt. IV *A*, Appendix 7, Rept. of U. S. Coast and Geodetic Survey, 1900, and Pt. IV *B*, Appendix 5, Rept. of U. S. Coast and Geodetic Survey, 1904.

[29] "Tides and Their Engineering Aspects", *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 713-715.

In conclusion, the writer wishes to express his admiration for the thoroughness of Colonel Brown's paper, and also his appreciation of the time and effort spent in its preparation.

EUGENE E. HALMOS,[30] M. AM. SOC. C. E. (by letter).[30a]—The writer has given careful attention to this paper, not only because of its general appeal to engineers interested in the science of hydraulics, but particularly because of the references made therein to the paper describing the Cape Cod Canal,[31] in the development of the mathematical portion of which the writer had the privilege of collaborating with Mr. Parsons.

The analysis of the tidal flow in canals presented in Mr. Parsons' paper followed the theory of Professor Maurice Lévy, which, in turn, is based on d'Alembert's theorem defining the interdependence of the free surface of a fluid in motion and the forces acting on its mass. The derivation of the formulas is strictly rational, their application being restricted only by the requirement of comparatively small tidal variations in relation to the depth of the canal.

In his study Colonel Brown starts with an equation (Equation (15)) which the writer considers empirical and in which the velocity of the current is assumed to be a very simple function of the speed of propagation, the depth of the canal, and the height of the tide at the summit of the wave. He then uses this value of the velocity in expressing the energy contained in the wave and the work done by friction in destroying this energy. From this empirical beginning, the author, by the application of well-established principles of hydrodynamics, and especially by the introduction of his very ingenious theory of the reflected wave from the reservoir, arrives at formulas and methods of treatment of the problem which are applicable to the varying conditions that may be encountered in tidal canals.

The author's treatment of the subject does not limit the use of his method and formulas to comparatively small tidal variations. It permits the consideration of friction in such a manner as to prove that its effect in modifying the height and the speed of the wave is greater for the negative than for the positive wave. This the writer believes to be an important advance in the study of tidal canals. The demonstration of the effect of interference of opposed waves is clear, instructive, and useful.

The author has shown that by his theory a very close check on both tidal heights and current velocity is obtained when applied to the Suez Canal. However, as stated by Mr. Parsons,[32] a similar agreement of computed and observed results is found with the Lévy formulas if the frictional coefficient is so modified that friction, as a linear function of the mean velocity, will absorb the same total energy as if its variation depended on the square of the velocity. On the other hand, the two methods seem to give appreciably different results in the case of more shallow canals with comparatively large tidal

[30] Chf. Engr., Parklap Constr. Corp., New York, N. Y.

[30a] Received by the Secretary, February 5, 1931.

[31] "The Cape Cod Canal," by William Barclay Parsons, Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII, (1918) p. 1.

[32] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 121.

variations. An inspection of Fig. 10 in the author's paper on "The Chesapeake and Delaware Canal"[33] and Fig. 17 and Fig. 19 of his present paper, at once reveals the fact that the tidal elevations computed by his method more closely approximate the observed profiles.

It appears that, while the mathematical basis of the analysis given in Mr. Parsons' paper is more rational, the results obtained from the formulas deduced by the author may conform better to actual observation for a wider range of conditions.

In the writer's opinion, the empiricism of Equation (15) does not seriously impair the validity of the author's formulas. He believes, however, that the method shown by Colonel Brown for the evaluation of the frictional coefficient, F, is of doubtful practical value. This method calls for the actual measurement of the velocity, V_0, at the outlet of the canal, which, of course, cannot be done before the canal is completed. For the design, therefore, this basic quantity is not available and must be evaluated by comparison with existing canals. As it is not likely that such canals, with closely similar cross-section, length, tidal conditions, and consistency of canal bed, could be easily found, the writer feels that it would be better to follow Mr. Parsons' suggestion and determine V_0 by one of the approximate methods given in his paper on the Cape Cod Canal.

Under "General Remarks", the author states that no theory can adequately and completely solve the complex problems which the regimen of the tide raises in practice and that such complete solution "must forever escape satisfactory analysis". In this, there is a complete agreement between the author and Mr. Parsons, who concluded that numerous necessary assumptions, which are never quite true in practical cases, "*a priori* reduce all analytical results to a grade of a more or less fair approximation". However, Mr. Parsons went further than the author when he definitely stated that, in his opinion, approximate methods of calculation should be used for the practical evaluation of the velocities in canals subjected to tidal influences.[34] On the basis of the theoretical studies, Mr. Parsons suggested the use of particular approximate formulas for different types of canals characterized by the product of their length and the factor, q, the value of which was given in his Equation (8).[35]

The writer found that this product for the Chesapeake and Delaware Canal amounts to 0.972 for the 11 miles between Mile 3 and Mile 14. It, therefore, would come under the Classification (c), as defined[36] in the paper on the Cape Cod Canal. For this type, Mr. Parsons recommended the use of the formulas of the French Academy of Sciences[37] as likely to give close results for practical purposes, both as to tidal elevations and maximum current velocities. The writer was unable to make a numerical check, by the use of these formulas, against the values computed by the author, because the published diagrams (Figs. 17 and 18) do not show the scale of either heights or velocities, nor could he check against observed current velocities as actual data are not yet available. The writer would suggest that the author make such a check when current

[33] *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 293.
[34] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 142.
[35] *Loc. cit.*, p. 112.
[36] *Loc. cit.*, p. 143.
[37] *Loc. cit.*, p. 138.

velocity measurements will furnish sufficient information on the Chesapeake and Delaware Canal and in this manner establish whether or not the recommendations made by Mr. Parsons as to the limits for the use of the proper approximate formulas need to be revised.

Nothing said by the writer in discussing this paper should detract from its value as a treatise of the highest order on the motion of water in tidal canals. The theory of the propagation of waves of translation in canals, conceived by Bourdelles and given mathematical expression by the author expands and completes the work of previous writers on the subject. In this paper, Colonel Brown has made a real and useful contribution to the science of hydrodynamics.

W. M. Black,[38] M. Am. Soc. C. E. (by letter).[38a]—The writer deems this paper of great value to the Engineering Profession. It is a distinct contribution to the very limited information heretofore available for determining in advance the movement of water in projected canals designed to connect bodies of water in which the tidal regimens differ. It will also assist in the determination of the effects of changes projected in the cross-sections of canals in which there is tidal action. From personal experience the writer knows how prone many engineers have been to treat tidal currents simply as water flowing under the laws which govern the uniform flow of water in pipes and channels under a fixed head. Too little weight has been given to the wave action propagated in advance of the current when the head in a stream is increased by an influx. McAlpine demonstrated this fact in the latter part of the Fifties and in the Nineties the late Brig.-Gen. Amos Stickney, U. S. A., reported the same phenomenon in his discussion of flood movements in the Mississippi River where, in the deep channel below the mouth of the Red River, the velocity of advance of the flood wave is five times greater than the maximum current velocity.

The velocity of wave propagation is greater than the velocity of the current and an increase of channel capacity increases the velocity of wave propagation and decreases the velocity of the induced current (Equations (2) and (16)). Therefore, the necessity of considering the changes of head due to the wave movement is apparent when the probable effect of changes in channel depths in waterways subject to tidal action is under study. The problem of the control of current velocities in the East River, New York, is an example.

Colonel Brown's theory seems to be supported in its application to the conditions in the several canals for which sufficient data are available. The writer believes that observations on other tidal canals will give further confirmation. He hopes that investigations along this line will be continued. He also hopes that similar analytical studies will be made of the tidal action in estuaries, such as the Hudson and Delaware Rivers, in which there is a gradual change in the form of the tidal wave from one of oscillation near the entrance from the sea to one of translation farther up the river.

[38] Maj.-Gen., U. S. A. (*Retired*), Washington, D. C.

[38a] Received by the Secretary, February 16, 1931.

It is only by a more accurate knowledge of the laws which govern the tidal wave action that the problems of the improvement of such estuaries can be solved with any degree of accuracy. Under the pressure of what seemed to be a local necessity, channel widths have been changed recently in the Lower Hudson. The effects of this change on the tidal regimen of the upper river cannot be foretold with any certainty at this time. The entire question of tidal action in inland waters is of the greatest practical importance.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THE DON MARTIN PROJECT

Discussion

BY C. H. HOWELL, M. AM. SOC. C. E.

C. H. HOWELL,[6] M. AM. SOC. C. E. (by letter).[6a]—The spillway of the Don Martín Dam is the first example of the round-head buttress type to be built. For this reason it may be interesting to give the reasons for its adoption.

Comparative designs were made for gravity, multiple-arch, and flat-slab or Ambursen sections. The foundation is stratified limestone. The strata are almost horizontal. The upper layers are separated by clay and shale seams. A worse condition for sliding can scarcely be imagined.

The gravity section proportioned to care for uplifting with these conditions was found to be exceedingly expensive. The possibility of unequal settlement of the buttresses, due to compression of the seam filling, with the attendant unknown stresses in the arches, eliminated serious consideration of the multiple-arch section. The flat-slab design, of course, was well adapted to meet the uplift and settlement conditions.

However, the round-head type was nearly as efficient as to uplift. Each unit being stable in itself it was equally efficient as to settlement. It required no reinforcing steel as all the material is in compression. There are no thin concrete sections exposed to water pressure. Due to the greater thickness and lower stresses of the up-stream face, a leaner concrete could be used in this portion than would be required for a flat-slab section. Also, due to the greater thickness, the operation of placing concrete is similar to the corresponding operation in a gravity dam. It is much cheaper than placing in the thin sections of the flat-slab design.

The comparative estimates, therefore, showed an appreciable saving in favor of the round-head buttress dam. Furthermore, it was and is considered to be structurally superior in that no tensile or bending stresses are involved.

NOTE.—The paper by Andrew Weiss, M. Am. Soc. C. E., was published in December, 1929, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

[6] Chf. Engr., J. G. White Eng. Corporation S. en C., Cit of Mexico, Mexico.

[6a] Received by the Secretary, February 9, 1931.

In regard to uplift on the base of the heads and buttresses the following five conditions were assumed:

Case 1.—No uplift.

Case 2.—Full reservoir head on one-half the area of the buttress head; and full tail-water head on one-half the area of the buttress stem.

Case 3.—Average of full reservoir head and full tail-water head on one-half the area of the buttress head; and full tail-water head on one-half the area of the buttress stem.

Case 4.—Average of full reservoir head and full tail-water head on entire area of the buttress head; and full tail-water head on the entire area of the buttress stem.

Case 5.—Full reservoir head on the entire area of the buttress head; and full tail-water head on the entire area of the buttress stem.

With the reservoir water surface at Elevation 858.70 (the top of the radial gates), computations indicated sliding coefficients as listed in Table 1 (Items 1 to 6, inclusive).

TABLE 1.—SLIDING COEFFICIENTS, DON MARTIN DAM

Item	Elevation	Case 1	Case 2	Case 3	Case 4	Case 5
WATER SURFACE AT ELEVATION 858.70						
1	844.21	0.323	.....	.....	.....	.....
2	809.50	0.597	0.762	0.670	0.762	1.052
3	787.20	0.575	0.692	0.628	0.692	0.868
4	770.80	0.583	0.677	0.627	0.677	0.809
5	754.40	0.591	0.684	0.642	0.702	0.812
6	740.62	0.588	0.682	0.646	0.721	0.820
WATER SURFACE AT ELEVATION 865.26						
7	844.21	0.618	.....	.....	.....	.....
8	809.50	0.756	1.002	0.863	1.002	1.484
9	787.20	0.665	0.810	0.730	0.810	1.038
10	770.80	0.654	0.765	0.705	0.765	0.922
11	754.40	0.640	0.750	0.704	0.783	0.907
12	740.62	0.622	0.735	0.696	0.789	0.894

The weight of concrete was assumed at 140 lb. per cu. ft. It actually weighs an average of 142 lb. Tail-water was assumed at Elevation 764. With the water surface at Elevation 865.26, or 6.56 ft. above the top of the radials, the coefficients are as shown in Table 1 (Items 7 to 12, inclusive). Tail-water was assumed at Elevation 770. The weight of the over-pour on the back slab was neglected.

If this dam were to be designed over again, "diamond heads" would be used instead of round heads. The same results are secured with a slight saving in concrete and somewhat easier form work. (See Fig. 12.)

At first, there was some doubt as to the amount of leakage which might occur between the units. There was some fear that it might be large. The reservoir water surface is now (January, 1931) about 10 ft. below the top of

the radials. To date, there has been no measurable leakage between the units. The copper seals between them have proved to be practically water-tight. Due to the excellent care used in construction there is no leakage through the horizontal joints between the various concrete lifts. A slight amount of moisture appeared at a few joints when the reservoir began filling, but these places have sealed themselves.

The concrete was approximately a 1:2.5:5 mix, and the coarse aggregate was crushed limestone passing a 3-in. screen. The sand was made of the same material, all of which passed a ¼-in. screen. The water-cement ratio was varied somewhat, but it averaged 1.20. The maximum length of a placement was about 100 ft., and the height of the lifts was 5 ft. The concrete was kept wet—not merely moist but wet—for a minimum of a week. The cement used on the project was manufactured near Monterrey, Mexico.

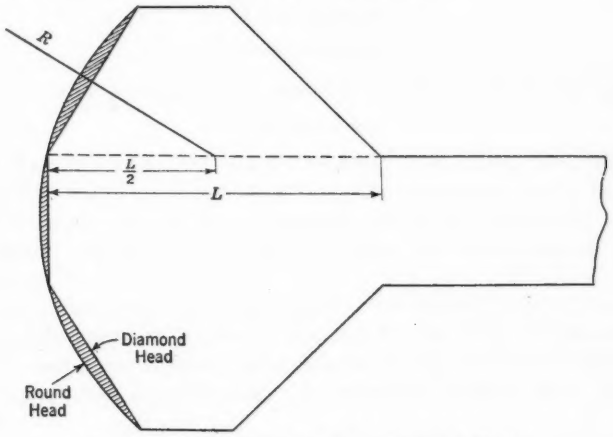

FIG. 12.—COMPARISON OF THE "DIAMOND HEAD" AND ROUND HEAD BUTTRESSES.

Practically no shrinkage trouble was experienced on this dam. In fact, practically no shrinkage has been noticed on any concrete placed on the entire project. This is believed to be due mostly to the extra care used in providing somewhat more than the usual quantity of curing water.

In a few of the buttresses hair cracks appeared in the crown of the arches in the lower gallery openings (Fig. 4). These cracks appeared during construction soon after the concrete was placed. They are vertical and are about 6 ft. long. After they were noticed, reinforcing steel was placed over the remaining openings. To date, no other cracks have been found. The existing cracks have not lengthened as the reservoir has filled.

As stated previously, the water surface in January, 1931, was about 10 ft. below the top of the radial gates. Therefore, actual service conditions had not been reached. However, the automatic action was tested by building a bulkhead in front of one gate and pumping the space between the bulkhead and gate full of water. This reproduced initial starting conditions exactly. Water was then pumped into the float-wells. Movement of the gate took place

almost exactly at the computed critical water depth in the float-well. The tests also indicated that an excess of power was available for operation, and that the gates would respond automatically to small changes in the reservoir water surface.

The writer believes that the round-head or diamond-head dam should be considered in all studies for sites where gravity, multiple-arch or flat-slab sections would be considered. He also believes that the round or diamond-head buttress dam has many inherent merits, and that these will become generally recognized in the future.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

FILTERING MATERIALS FOR WATER AND SEWAGE WORKS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION

Discussion

BY MESSRS. W. S. MAHLIE, AND O. J. RIPPLE

W. S. MAHLIE,[1] ESQ. (by letter).[1a]—After comparing the results of the sodium sulfate tests on limestone, it appears that there might be a general indication as to the suitability of material for filters. However, this is only apparent when considering the results from the entire group of testing laboratories as "mass data".

There is considerable variation between the laboratories represented, and even large differences on duplicate samples by the same laboratories. It is pointed out, that some of them made changes in the recommended method, but even those reported as following the procedure closely, show rather wide variations.

If an engineer were selecting the material on a laboratory analysis basis, he probably would have only one laboratory's results, possibly two, but hardly as many as are indicated in the report. In view of the wide variation, it is doubtful whether he would be able to form an opinion strictly on numerical values.

While the results of the study present evidence that filtering material can be judged by a sulfate test, the writer is somewhat inclined to think that if the results of one laboratory were to be the only ones considered, there would be some hesitancy in using the loss of weight as an index of quality. Taking perhaps ten stones as representing the sample and using the average loss, might possibly offer some closer agreement.

In reading the section entitled "Theory of the Sodium Sulfate Test", it appears that the disruptive effect is ascribed to the absorption of sodium

NOTE.—The Progress Report of the Committee of the Sanitary Engineering Division on Filtering Materials for Water and Sewage Works, was presented at the meeting of the Division, New York, N. Y., January 16, 1930, and published in October, 1930, *Proceedings*. Discussion of the report has appeared in *Proceedings*, as follows: October, 1930, by Messrs. James W. Armstrong and W. H. Dittoe.

[1] Chemist in Chg., Water and Sewage Purification, Water Dept., Fort Worth, Tex.

[1a] Received by the Secretary, December 11, 1930.

sulfate and a subsequent crystallization on drying. This leads one to wonder whether it would be possible to measure the absorption of water in some manner, and correlate it with other results, rather than go through the tedious twenty cycles of immersion and drying.

These views, of course, are presented without any experience in the actual testing and are merely an expression of opinion based on reading the report. Apparently, there has been quite a great deal of work involved in making these tests, but it is unfortunate that several of the laboratories either departed from, or interpreted, the method differently.

The various contributing workers are to be commended for the great amount of work they have done, and especially those who applied the tests from 100 to 250 cycles.

O. J. RIPPLE, ESQ.[2] (by letter).[2a]—The Marston Lake North Side Filter Plant at Littleton, Colo., supplied some of the data included as a part of this report. The experimental filters used were 2-in. glass tubes, 5 ft. long, except Nos. 7 and 9, in Table 10, which were 7 ft. long, to take care of the extra depth of the anthracite coal which is used for the filter material in this plant. The tubes had an inside diameter of 1.88 in.

Two runs were made on somewhat different plans of operation. The first run was eight months long, from May to December, 1928, inclusive, using a 40-in. rate of wash for 3 min. for the sand filters and a 24-in. rate of wash for 5 min. for the coal filters. In both runs Filter No. 8 was the large unit to which the experimental filters were attached. Bacteria and micro-organism samples were taken during the first run, but were not taken through the second run, because of the poor bacterial removal during the first run, which was attributed to difficulty in washing the small filters.

TABLE 10.—CHARACTERISTICS OF EXPERIMENTAL FILTERS AT DENVER, COLO.

Filter No.	Depth, in inches	Material	Effective size, in millimeters	60% size	Uniformity coefficient
1	24	Sand	0.36	0.64	1.76
2	24	Sand	0.32	0.53	1.67
3	24	Sand	0.61	0.75	1.23
4	30	Sand	0.34	0.62	1.82
5	24	Sand	0.40	0.58	1.45
6	20	Sand	0.41	0.57	1.39
7	47	Coal	0.55	1.10	2.00
8	47	Coal	0.56	1.18	2.05
9	24	Coal (top layer)	0.83	....	1.1
	15	Sand (bottom layer)	0.50	....	1.3

During the second run of seven months, from June to December, 1929, inclusive, different rates of wash were used for each filter, the wash being continued until the turbidity of the wash water was approximately 70 parts per million. From the results listed in Table 10, it will be noted that the percentage of wash water, during the second run, is less for about the same length of runs.

[2] Supt., Marston Lake North Side Filter Plant, Denver, Colo.

[2a] Received by the Secretary, February 2, 1931.

The six samples of sand sent by the Committee and used in Filters Nos. 1 to 6, inclusive, were measured as so many inches of the various sieve sizes and not by effective size and uniformity coefficient as is customary in designating filter materials. However, in Table 10, the approximate rating by effective size and uniformity coefficient is given, because they will mean more to the operators expressed in these terms.

TABLE 11.—Data Obtained from Two Runs, Experimental Filters at Denver, Colo.

Filter No.	Number of runs	Material	Average length of run, in hours	Percentage of wash water	Rate of wash, in inches per minute	Rise sand, inches
			First Run			
1	165	Sand	17.7	4.55	40.	13.1
2	196	Sand	7.1	10.79	40.	24.5
3	139	Sand	24.8	3.16	40.	11.7
4	190	Sand	12.1	7.20	40.	20.5
5	190	Sand	11.2	7.86	40.	24.6
6	188	Sand	11.9	6.95	40.	17.5
7	109	Coal	33.5	2.44	24.	20.2
8	183	Coal	26.3	2.47	18.	14.5
			Second Run			
1	134	Sand	17.4	2.48	50.6	23.4
2	134	Sand	8.1	4.74	36.7	23.9
3	128	Sand	23.6	1.96	55.2	21.9
4	134	Sand	11.9	2.71	31.3	18.1
5	134	Sand	10.6	3.58	37.9	24.4
6	134	Sand	10.8	3.47	43.3	25.2
7	113	Coal	29.6	1.67	28.4	25.9
8	126	Coal	35.9	1.88	17.7	14.3
9	67	.....	50.4	1.28	31.4	36.7

Filter No. 9 was a combination of sand and coal and was not run during the first period of operation.

TABLE 12.—Analysis of Raw Water

Test	First Run			Second Run		
	Maximum	Minimum	Monthly average	Maximum	Minimum	Monthly average
Turbidity, in parts per million.	8.3	4.9	6.7	12.6	3.2	7.2
Temperature, in degrees centigrade	19.8	3.2	10.4	20.4	3.0	10.2
Akalinity, in parts per million:						
pH	8.42	7.85	8.22	8.11	7.78	7.96
MO	92	67	78	78	71	75
Phenol	5	1	3	1	0	0.5
Bacteria per cubic centimeter.	521	208	334	585	53	264
Micro-organisms in standard units per cubic centimeter.	1757	3.05	656	165.	344	675
Chemicals, in grains per gallon:						
Alum	0.667	0.123	0 254	0.443	0.168	0.256
Dry sodium aluminate	0.133	0.075	0.113	0.114	0.049	0.083

Table 11 gives some of the data regarding the two runs. It will be noted that the coarsest sand filter (No. 3) gave the best results of any of the sand filters, also, that No. 9, a combination of sand and coal, gave results better than either the sand or coal used separately.

The raw water treated during these two runs was characterized by a low turbidity, as shown by Table 12. This is due to its storage in Marston Forks, which has a capacity of 6 500 000 000 gal.